Graphs, Games, and Pragmaticism’s Proof

Ahti-Veikko Pietarinen

Department of Philosophy
University of Helsinki
ahti-veikko.pietarinen@helsinki.fi

December 2005
Outline

Diagrammatic Logic of Existential Graphs

Semantics: Games

The Proof of Pragmaticism
Charles Sanders Peirce (1839–1914)

*I do not think I ever reflect in words: I employ visual diagrams, firstly, because this way of thinking is my natural language of self-communion, and secondly, because I am convinced that it is the best system for the purpose.* (MS 619, 1909).

Peirce’s goal was a logical analysis of thought and reasoning that is rigorous and valid also when symbolic expressions fall short of fulfilling that purpose.

*There are countless Objects of consciousness that words cannot express; such as the feelings a symphony inspires or that which is in the soul of a furiously angry man in [the] presence of his enemy.* (MS 499, 1906).

No one has invented logical diagrams for feelings, but Peirce strongly believed in their plausibility.
“Moving Pictures of Thought”

According to Peirce, graphical representation of natural language puts before us

- “moving pictures of thought” (CP 4.11)
- “a moving picture of the action of the mind in thought” (MS 298: 1)
- “a portraiture of Thought”.

The precise vehicle is the iconic logic of diagrams, which will

- “furnish a moving picture of the intellect” (MS 298: 10) and provides a “system for diagrammatizing intellectual cognition” (MS 292: 41).
Diagrammatic Logic

What is essential here?

- **Iconic** representations: denote things represented by likeness, semblance, analogy (**graphs, diagrams, models, sets of sentences, ...**).
- May be abstract, structural, intellectual likeness (‘true-in-a-model’, (homo/auto)morphisms, structure-preserving maps, ...)
- Modern incarnation: Conceptual Graphs (CG) in Computer Science & AI, a ‘semantic web’, ...
- The original formulation was in terms of the very expressive system of *Existential Graphs* (EGs, 1896).
Image, Diagram, Metaphor

A Diagram is a [sign] which is predominantly an icon of relations and is aided to be so by conventions. Indices are also more or less used. (MS 492: 22).

A diagram should be “as iconic as possible” in order to represent “visible relations” (MS 492: 22). Nowadays there are the heterogeneous logics that are not fully iconic.

Not all iconicity is diagrams, however. Iconic signs (hypoicons) fall into three classes:

Those which partake of simple qualities, or First Firstnesses, are images; those which represent the relations, mainly dyadic, or so regarded, of the parts of one thing by analogous relations in their own parts, are diagrams; those which represent the representative character of a representamen by representing a parallelism in something else, are metaphors. (EP 2:273, 1903).

So iconic logic should really embody also images and metaphors.
A Recap

idea
↑
Vorstellung
↑
representation
↑
Icon ← Index ← Symbol
↑
Image ← Diagram ← Metaphor
↑
Graph
↑
Logical Graph
↑
Existential Graph (Alpha, Beta, Gamma)

I extend logic to embrace all the necessary principles of semeiotic, and I recognize a logic of icons, and a logic of indices, as well as a logic of symbols. (CP 4.9, 1906).
Alpha, Beta, Gamma

“Very expressive”:

1. Alpha Graphs \(\sim\) propositional logic
   \[\text{Cat and dog are on a mat.}\]
2. Beta Graphs \(\sim\) predicate logic
   \[\text{Every man is mortal.}\]
3. Gamma Graphs \(\sim\)
   3.1 Modal logic
      \[\text{It is possible that it rains.}\]
   3.2 Higher-order logic
      \[\text{Aristotle has all the virtues of a philosopher.}\]
   3.3 Metagraphs
      \[\text{‘You are a good goalkeeper’ is much to be said.}\]
   3.4 Non-declaratives
      \[\text{interrogatives, imperatives, emotions, interpretation of music...}\]
Alpha Graphs

Definition
The set of Alpha Graphs $\mathcal{G}_\alpha$ is the smallest set of satisfying:

1. *Sheet of Assertion (SA)*: $\square \in \mathcal{G}_\alpha$.
2. Closure under *juxtaposition*:
   
   If $P_1 \in \mathcal{G}_\alpha$, $P_2 \in \mathcal{G}_\alpha \ldots P_n \in \mathcal{G}_\alpha$, then $[P_1 \ldots P_n] \in \mathcal{G}_\alpha$.
3. Closure under *cuts*:
   
   If $P_1 \in \mathcal{G}_\alpha$, then $\bigcirc P_1 \bigcirc \in \mathcal{G}_\alpha$.

Remark

- Cuts may not overlap.
- Juxtaposition is *commutative* and *associative*.
Alpha Graphs

EGs are scribed on a surface.

1. **Sheet of Assertion**

   ![Diagram of Sheet of Assertion]

   T (verum)

2. **Juxtaposition**

   conjunction

3. **Cut** ~ negation
Alpha Graphs

EGs are scribed on a surface.

1. **Sheet of Assertion**

![T (verum)](image)

2. **Juxtaposition ~**

*conjunction*

*Pear is ripe*

3. **Cut ~** negation
Alpha Graphs

EGs are scribed on a surface.

1. Sheet of Assertion

   \[ T \text{ (verum)} \]

2. Juxtaposition \&

   conjunction

   *Pear is ripe*
   
   *A dog stumbles over a quick fox*

   \[ p \land q \]

3. Cut \& negation
Alpha Graphs

EGs are scribed on a surface.

1. **Sheet of Assertion**
   
   \[ T \text{ (verum)} \]

2. **Juxtaposition** ∼
   
   conjunction

   \[ \text{Pear is ripe} \]
   
   \[ A \text{ dog stumbles over a quick fox} \]

   \[ p \land q \]

3. **Cut** ∼ negation
   
   \[ \text{You are cautious} \]

   \[ p \]
Alpha Graphs

EGs are scribed on a surface.

1. **Sheet of Assertion**
   
   ![Diagram](image)

   \[ T \text{ (verum)} \]

2. **Juxtaposition** ~
   
   **conjunction**
   
   ![Diagram](image)

   \[ \text{Pear is ripe} \]
   \[ A \text{ dog stumbles over a quick fox} \]

   \[ p \land q \]

3. **Cut** ~ negation
   
   ![Diagram](image)

   \[ \neg p \]
Alpha Graphs

EGs are scribed on a surface.

1. **Sheet of Assertion**

   \[ \mathsf{T} \ (\text{verum}) \]

2. **Juxtaposition** \sim \ 

   conjunction

   \[ \text{Pear is ripe} \]
   
   \[ A \text{ dog stumbles over a quick fox} \]

   \[ p \land q \]

3. **Cut** \sim \ 

   negation

   \[ \neg p \]

   \[ \text{You are cautious} \]
   
   \[ \text{Pear is ripe} \]
   
   \[ A \text{ dog stumbles over a quick fox} \]
Alpha Graphs

EGs are scribed on a surface.

1. **Sheet of Assertion**
   - $\top$ (verum)

2. **Juxtaposition** $\sim$
   - conjunction
     - **Pear is ripe**
     - **A dog stumbles over a quick fox**
     - $p \land q$

3. **Cut** $\sim$ negation
   - $\neg p$
   - **You are cautious**
   - **Pear is ripe**
   - **A dog stumbles over a quick fox**
   - $\neg (p \land q)$
Alpha Graphs

EGs are scribed on a surface.

1. Sheet of Assertion

   T (verum)

2. Juxtaposition

   Pear is ripe
   A dog stumble over a quick fox

   $p \land q$

3. Cut ~ negation

   $\neg p$

   You are cautious

   Pear is ripe
   A dog stumble over a quick fox

   $\neg (\neg p \land \neg q) = p \lor q$
Alpha Graphs

EGs are scribed on a surface.

1. **Sheet of Assertion**
   \[ \top (\text{verum}) \]

2. **Juxtaposition** \( \sim \)
   conjunction
   \[ \begin{align*}
   \text{Pear is ripe} \\
   \text{A dog stumbles over a quick fox}
   \end{align*} \]
   \[ p \land q \]

3. **Cut** \( \sim \) **negation**
   \[ \neg p \]
   \[ \text{You are cautious} \]
   \[ \neg (\neg p \land \neg q) = p \lor q \]
   \[ \text{Pear is ripe} \]
   \[ \text{A dog stumbles over a quick fox} \]
   \[ p \land q \]
   \[ \neg (p \land \neg q) = \neg p \lor q = p \rightarrow q \]
Alpha Graphs

EGs are scribed on a surface.

1. Sheet of Assertion
   \[ T \text{ (verum)} \]

2. Juxtaposition \(\sim\) conjunction
   \[ p \land q \]

3. Cut \(\sim\) negation
   \[ \neg p \]
   \[ \text{You are cautious} \]
   \[ \neg(p \land \neg q) = p \lor q \]
   \[ \text{Pear is ripe} \]
   \[ A \text{ dog stumbles over a quick fox} \]

   \[ \neg(p \land \neg q) = \neg p \lor q = p \rightarrow q \]
   \[ (\text{the scroll}) \]

   \[ \neg T = \bot \text{ (falsum)} \]
Alpha Graphs

Definition (Area)
Space within the cut without the cut is the area of the cut.

Definition (Enclosure)
A cut, its area and everything in that area comprise the enclosure of the cut.

Remark
1. Area is not part of the SA.
2. Area is not a graph.
3. Cut is not a graph.
4. Enclosure is a graph.
Alpha Graphs

Definition (Positive and negative areas)

1.a Any graph $P \in \mathcal{G}_\alpha$ not enclosed by any cut or enclosed by an even number of cuts is *evenly enclosed*.

b Any graph $P \in \mathcal{G}_\alpha$ enclosed by an odd number of cuts is *oddly enclosed*.

2.a Area on which an evenly enclosed graph rests is *positive*.

b Area on which an oddly enclosed graph rests is *negative*.

Remark
The union of evenly and oddly enclosed graphs of any $P \in \mathcal{G}_\alpha$ comprise the set of all subgraphs of $P$. 
Alpha Graphs

Definition (Nest)

- A linearly ordered finite sequence of areas from the SA to the areas of cuts of increasing depth makes a *nest*.
- A nest terminating on a cut-free area is a *maximal nest*.

\[ A \]

1. One of 5 areas, or 4 cuts A-B-C-E-F
   Three of 4 areas or 3 cuts each:
   2. A-B-C-D
   3. A-B-H-I
   4. A-B-H-J
   5. One of 3 areas, or 2 cuts, A-B-G.
Five rules of transformation:

1. **Add/remove double cuts:**

   \[ \cdots P \quad Q \quad \Leftrightarrow \quad \cdots P \quad Q \]

2. **Insertion:** Any \( P \in G_\alpha \) may be added on negative area:

   \[ \cdots 2k+1 \quad k \quad \Rightarrow \quad \cdots P \quad 2k+1 \quad k \]

3. **Erasure:** Any \( P \in G_\alpha \) may be erased from positive area:

   \[ \cdots P \quad 2k \quad k \quad \Rightarrow \quad \cdots 2k \quad k \]
Proofs

4. **Iteration**: Any copy of $P$ may be **scribed on** the same area or on the area in its nest (not part of $P$):

   $\cdots P \cdots \implies \cdots PP \cdots$

5. **Deiteration**: Any copy of $P$ may be **removed** from the same area or from the area in its nest (not part of $P$):

   $\cdots P^P \cdots \implies \cdots P \cdots$
Proofs

“Moving pictures of thought”: 
Beta Graphs

Definition (Beta Graphs)
The set of beta graphs $\mathcal{G}_\beta$ is the smallest set satisfying:

1. **SA**: $\square \in \mathcal{G}_\beta$.

2. **Dot**, **Line of Identity** (LI) $\square \quad \square \in \mathcal{G}_\beta$ and finitely branching LI $\square \quad \square \in \mathcal{G}_\beta$.

3. **Closure under spots**:
   - If $\bullet$, $\square \quad \square$, and $\square \quad \square \in \mathcal{G}_\beta$, then $\bullet \quad \square \quad \square \quad \square \in \mathcal{G}_\beta$.

4. **Closure under juxtaposition**:
   - If $\varphi_1 \in \mathcal{G}_\beta \ldots \varphi_n \in \mathcal{G}_\beta$, then $\varphi_1 \ldots \varphi_n \in \mathcal{G}_\beta$. 
Beta Graphs

Definition (cont.)

5. Closure under cuts:

If \[ \begin{array}{c}
\bullet \\
\end{array}, \begin{array}{c}
- \\
\end{array}\] and \[ \begin{array}{c}
\text{\textbullet} \\
\end{array}\] \(\in \mathcal{G}_\beta\), then

\[ \begin{array}{c}
\circ \end{array}, \begin{array}{c}
- \\
\end{array}, \begin{array}{c}
- \\
\end{array}\]
and \[ \begin{array}{c}
\text{\textbullet} \\
\end{array}\] \(\in \mathcal{G}_\beta\).

Remark

- SA represents the universe of discourse.
- Cuts may not cross spots.
- LLs are finitely long.

Definition (Outer end)
The least enclosed portion of an LL is its outermost end.
4. $\mathbf{L1} \sim$ quantification

- $\exists x \ (x = x)$
Beta Graphs

4. $L_1 \sim \text{quantification}$

$$\exists x \exists y \ (x = y)$$
Beta Graphs

4. $\exists x \, R(x)$

\[
\begin{array}{c}
\text{R} \\
\exists x \, R(x)
\end{array}
\]
Beta Graphs

4. LI $\sim$ quantification

$\exists x \exists y \, R(x, y)$

*Someone loves someone.*
Beta Graphs

4. LI \sim quantification

\[ \exists x \exists y \, R(x, y) \]

*Someone loves someone.*

\[ \exists x \exists y \, (LC(x) \land CD(y) \land (x = y)) \]

*Lewis Carroll is Charles Dodgson.*
4. LI \sim quantification

\[ \exists x \exists y \ R(x, y) \]

\textit{Someone loves someone.}

\[ \exists x \exists y \ (L(x) \land C(y) \land (x \neq y)) \]

\textit{Lewis Carroll is not Charles Dodgson.}
Beta Graphs

4. LI ~ quantification

\[ \exists x \exists y \ R(x, y) \]

*Someone loves someone.*

\[ \exists x \exists y \ (L C(x) \land C D(y) \land (x \neq y)) \]

*Lewis Carroll is not Charles Dodgson.*
Beta Graphs

4. LI \sim quantification

\[ R \quad \exists x \exists y \ R(x, y) \]

*Someone loves someone.*

\[ \exists x \exists y \ (L(x) \land C(y) \land (x \neq y)) \]

*Lewis Carroll is not Charles Dodgson.*
Beta Graphs

4. LI \sim quantification

\[ \exists x \exists y \, R(x, y) \]
\[ R \]

Someone loves someone.

\[ \exists x \exists y \, (LC(x) \land CD(y) \land (x \neq y)) \]
Lewis Carroll is not Charles Dodgson.

- **Existential assertion**: The outermost end of an LI rests on a positive area.
- **Universal assertion**: The outermost end of an LI rests on a negative area.
Beta Graphs

Definition (Loose End)
Unattached end of an LI is a *loose end*.

- Any hook may be attached by an LI.
- At most one LI may be attached to a hook.
- Attachment corresponds to a bound variable (no free variables).
- LIs whose outermost loose ends lie within the same area form an *equivalence class*.

Definition (Ligature)
A line that branches or crosses a cut is a *ligature*.

- LIs may not cross cuts, ligatures may.
- Ligatures $\notin \mathcal{G}_\beta$

Definition (Cycle, Bridge)

- A self-returning LI is a *cycle*.
- LIs that cross but do not join form a *bridge*. 
Examples

There is a woman who loves (and is loved by) every man:

Every man loves (and is loved by) a woman:

(For simplicity, assume that all relations are symmetric).
Shin’s Account


1. Shin’s interpretation is not as iconic as one wishes for (e.g., uses predicate terms, so in fact amounts to heterogeneous logics).
2. Misses non-diagrammatic aspects of icons; no Gamma.
3. Idiosyncratic on “visually clear” and “intuitive” ways of how one could “read off” EGs.
4. There are always multiple ways – this is not an issue in EGs.
Semantic Games

A semantic game $G$ is played according to a graph $\varphi \in G_\beta$ on a model $M = \langle D, I \rangle$, in which $D$ is the universe of discourse and $I$ interpretation. $G(\varphi, M)$ is a perfect-information zero-sum game between GRAPHIST and GRAPHEUS.

1. **Juxt-rule:**
   
   1.1 *Juxtaposition on positive area:* GRAPHIST chooses one subgraph in $\varphi$. $G(\varphi, M)$ goes on according to that choice.
   
   1.2 *Juxtaposition on negative area:* GRAPHEUS chooses one subgraph. $G(\varphi, M)$ goes on according to that choice. *Winning conventions will change.*

2. **Lig-Rule:**

   2.1 *Outermost end on positive area:* GRAPHIST chooses an element from $D$ and attaches its name to that end. $G(\varphi, M)$ goes on according to that choice. Attachment to hooks is denoted by a dot and the ligature is removed.

   2.2 *Outermost end on negative area:* GRAPHEUS chooses an element from $D$. . .
Semantic Games

3. **Winning conventions:** When a spot $S$ is reached, its value determines the winner of a play of a $G(\varphi, M)$:
   
   3.1 If $S$ is true, GRAPHIst wins the play (payoff $(1, -1)$).
   3.2 If $S$ is not true, GRAPHEUS wins the play (payoff $(-1, 1)$).

4. **Winning rule:** The existence of a winning strategy in $G(\varphi, M)$ determines the truth-value of $\varphi \in G_\beta$ in $M$:
   
   4.1 $\varphi$ is true in $M$ iff there exists a w.s. for GRAPHIst in $G(\varphi, M)$.
   4.2 $\varphi$ is false in $M$ iff there exists a w.s. for GRAPHEUS in $G(\varphi, M)$.

[The Graphist is] the author of truth (for we have seen that falsity is what he forbids and truth what he permits). (MS 280: 29)

The reason why it is necessary to imagine a Graphist as well as an interpreter [The Grapheus] is [that] logic cannot be successfully studied without perfectly clear ideas. Now the graphs and the sheet of assertion are represented as signs; but if they are signs, they must, according to the principles of pragmaticism, function as such. For it will be found to be a corollary from that principle that existence consists in action. (MS 280: 29-30)
Semantic Games

3. **Winning conventions**: When a spot $S$ is reached, its value determines the winner of a play of a $G(\varphi, M)$:
   
   3.1 If $S$ is true, **GRAPHIST** wins the play (payoff $(1, -1)$).
   
   3.2 If $S$ is not true, **GRAPHEUS** wins the play (payoff $(-1, 1)$).

4. **Winning rule**: The existence of a winning strategy in $G(\varphi, M)$ determines the truth-value of $\varphi \in \mathcal{G}_\beta$ in $M$:
   
   4.1 $\varphi$ is *true* in $M$ iff there exists a w.s. for **GRAPHIST** in $G(\varphi, M)$.
   
   4.2 $\varphi$ is *false* in $M$ iff there exists a w.s. for **GRAPHEUS** in $G(\varphi, M)$.

-  
  [The Graphist is] the author of truth (for we have seen that falsity is what he forbids and truth what he permits). (MS 280: 29)

- The reason why it is necessary to imagine a Graphist as well as an interpreter [The Grapheus is] that logic cannot be successfully studied without perfectly clear ideas. Now the graphs and the sheet of assertion are represented as signs; but if they are signs, they must, according to the principles of pragmaticism, function as such. For it will be found to be a corollary from that principle that existence consists in action. (MS 280: 29-30)
Example

Here is an extensive-form game $G(\varphi, M)$ for a Beta Graph $\varphi = \text{Every man loves (and is loved by) a woman}$, in which

$M = \langle D, I \rangle$

$D = \{\text{John, Mary} \ldots \}$

$I(a) = \text{John}, I(b) = \text{Mary,} \ldots$

$I(\text{man}) = \{\text{John,} \ldots \}$

$I(\text{woman}) = \{\text{Mary,} \ldots \}$

$I(\text{loves}) = \{(\text{John, Mary}), (\text{Mary, John}) \ldots \}$.
Games & Peirce

Peirce defined the meaning of intellectual concepts in terms of interaction between the Utterer and the Interpreter. This he did also for quantifiers $\Pi$ and $\Sigma$:

\[ [\Pi \text{ and } \Sigma] \text{ show whether the individuals are to be selected universally or existentially, that is, by the interpreter or by the utterer. (MS L 107: 8, 1905).} \]

In the sentence “Every man dies,” “Every man” implies that the interpreter is at liberty to pick out a man and consider the proposition as applying to him. (CP 5.542, c.1902).

Universal (Any) and particular assertions (Some) are duals:

Instead of the selection of the instance being left — as it is, when we say “any man is not good” — to the opponent of the proposition, when we say “some man is not good,” this selection is transferred to the opponent’s opponent, that is to the defender of the proposition. Repeat the same, and the selection goes to the opponent’s opponent’s opponent, that is, to the opponent again, and it becomes equivalent to any. (CP 3.481)
Games & Peirce

Consider

“Any man will die,” allows the interpreter, after collateral observation has disclosed what single universe is meant, to take any individual of that universe as the Object of the proposition, giving, in the above example, the equivalent “If you take any individual you please of the universe of existent things, and if that individual is a man, it will die”. (Essential Peirce 2:408, 1907)

Cf. the interpretation that Game-Theoretic Semantics assigns to any:

*If the game has reached the sentence*

\[ X \land Y \text{ who } Z \land W, \]

*then Nature [= Interpreter] may choose an individual and give it a proper name (if it did not have one already), say ‘b’. The game is continued with respect to*

\[ X \land b \land W, \text{ b is } a(n) \text{ Y, and (if) } b \text{ Z}. \] (Hintikka 1979)
Games & Peirce

The action requires a source of concepts to be conveyed, and therefore in some sense a mind from which the concepts, propositions, and arguments are conveyed to the mind of the interpreter; and the two minds must be capable of coming to an understanding and of observing it when it is reached. This supposes a power of deliberate self-controlled thinking. Now nothing can be controlled that cannot be observed while it is in action. It is therefore requisite that both minds but especially the Graphist-mind should have a power of self-observation. Moreover, control supposes a capacity in that which is to be controlled of acting in accordance with definite tendencies of a tolerably stable nature, which implies a reality in this governing principle. But these habits, so to call them, must be capable of being modified according to some ideal in the mind of the controlling agent; and this controlling agent is to be the very same as the agent controlled; the control extending even to the modes of control themselves, since we suppose that the interpreter[-mind] under the guidance of the Graphist-mind discusses the rationale of logic itself. (MS 280: 30–32, The Basis of Pragmaticism, 1905)
Games & Peirce

- “the two minds” (Utterer/Graphist–Interpreter/Grapheus) = Verifier–Falsifier
- “capable of coming to an understanding and of observing it” = payoffs
- “self-controlled thinking”, “self-observation” = strategic thinking
- self-control + “definite tendencies of a tolerably stable nature” = winning strategies, habits of acting for a purpose.

Habits have a counterfactual nature:

“Now, the identity of a habit [= strategy] depends on how it might lead us to act, not merely under such circumstances as are likely to arise, but under such as might possibly occur, no matter how improbable they may be. (No matter if contrary to all previous experience). (CP 5.400, 1877/93, How to Make Our Ideas Clear)

This is how strategies are conceived in game theory, too.
Habits

The essential function of a sign is to render inefficient relations efficient, ... not to set them into action, but to establish a habit or general rule whereby they will act on occasion. (CP 8.332, 1904, Letter to Lady Welby)

Pragmaticism concerns establishing habits as interpretants arising in self-controlled action.

Under given conditions, the interpreter will have formed the habit of acting in a given way whenever he may desire a given kind of result. The real and living logical conclusion is that habit; the verbal formulation merely expresses it. ... But action cannot be a logical interpretant, because it lacks generality. ... Consequently, the most perfect account of a concept that words can convey will consist in a description of the habit which that concept is calculated to produce. But how otherwise can a habit be described than by a description of the kind of action to which it gives rise, with the specification of the conditions and of the motive? (CP 5.491, 1906-07, Survey of Pragmaticism; MS 318).
The Proof: Background

Peirce said that he has “an ample supply of seductive persuasions to pragmatism”; “two or three scientific proofs of its truth” (CP 5.468, 1907). Moreover:

*If I may trust my most cautious logic, such a proof I have worked out and perfected. “What is it, then? Produce it.” Unfortunately, like many another intricate proof[s], it only becomes evident upon close, severe, and long study. (MS 322, Feb/March 1907)*

*I have replaced by a scientific and logical proof the merely rhetorical defence I made of the principle in my two original articles in the Popular Science Monthly of Nov. 1877 and Jan. 1878. (MS 296, March 1908)*

What is this proof? How to reconstruct it?
The Proof: Background

We relate Peirce’s proof of pragmaticism to verificationistically interpreted game-theoretic conception of meaning. At the bottom lies a model-theoretic and calculistic analysis of language:

1. Application of game theory provides a persuasive way to understand the notion of truth.
2. Language is a re-interpretable system.
3. Semantics is effable, in other words the relationship between language and the world can be articulated and theorised about by the use and application of language.

Peirce’s role model was that of a practising laboratorian, who takes the meaning of experiments to be in the results that they produce.
The Proof

The “Kernel” of Pragmaticism:

*The total meaning of the predication of an intellectual concept consists in affirming that, under all conceivable circumstances of a given kind, the subject of the predication would (or would not) behave in a certain way,—that is, that it either would, or would not, be true that under given experiential circumstances ... certain facts would exist.* (EP 2:402)

1. Every thought, intellectual concept and generality (*tig*) is a sign.
   1.1 If thinking is dialogue between the utterer and the interpreter, every *tig* is a sign.
   1.2 Thinking is dialogue between the utterer and the interpreter.

2. Something is a sign, iff it is possible for it to have the essential ingredients (*quaesitum*) of the utterer and the interpreter.

3. A sign can have the essential ingredients of the utterer and the interpreter (*quaesitum*), iff it can have an object (*requaesitum*).
The Proof

1. Every thought, intellectual concept and generality (‘tig’) is a sign.
   1.1 If thinking is dialogue between the utterer and the interpreter, every tig is a sign.
   1.2 Thinking is dialogue between the utterer and the interpreter.
2. Something is a sign, iff it is possible for it to have the essential ingredients (quaesitum) of the utterer and the interpreter.
3. A sign can have the essential ingredients of the utterer and the interpreter (quaesitum), iff it can have an object (requaesitum).
4. A sign has an object, iff the individuals that constitute the extension of the sign can be picked in a semantic game.
5. The individuals constituting the extension of the sign can be picked in a semantic game only if the sign has an interpretant.
6. Hence, by definition something is a sign, iff it mediates between the object and the interpretant and it has an object through the interpretant and an interpretant determined by the object.
The Proof

7. All $\text{tig}$-signs give their objects as their contribution to the purpose that can be reached by them.

8. We can acquire an object only by making experiments on various ways to find solutions in our thoughts.

9. All $\text{tig}$-signs give their objects through habits that we have received through experiments made in our thoughts.

10. $\rightarrow$ Final logical interpretants of $\text{tig}$-signs are habits that we have received through experiments made in our thoughts.

- “The kernel”: the whole meaning of a $\text{tig}$-sign is that “certain kinds of events would happen, once in so often, in the course of experience, under certain kinds of existential circumstances”.

- In other words, the final logical interpretants of $\text{tig}$-signs, their meanings, are habits of acting in a certain way in certain circumstances, which is to say are mappings from situations to actions.
The Proof

7. All *tig*-signs give their objects as their contribution to the purpose that can be reached by them.

8. We can acquire an object only by making experiments on various ways to find solutions in our thoughts.

9. All *tig*-signs give their objects through habits that we have received through experiments made in our thoughts.

10. → Final logical interpretants of *tig*-signs are habits that we have received through experiments made in our thoughts.

▶ “The kernel”: the whole meaning of a *tig*-sign is that “certain kinds of events would happen, once in so often, in the course of experience, under certain kinds of existential circumstances”.

▶ In other words, the final logical interpretants of *tig*-signs, their meanings, are habits of acting in a certain way in certain circumstances, which is to say are mappings from situations to actions.
The Proof, in GTS

1. A graph $\varphi$ is true in $M$ iff there exists a winning strategy for the Graphist in a semantic game $G(\varphi, M)$.

2. The Graphist has a winning strategy in $G(\varphi, M)$ iff there exists a habit of action associated with $\varphi$ by which we can choose suitable courses of actions and seek and find suitable elements from our universes of discourse.

3. Juxtaposition, polarities of areas and continuous connections between subspaces contribute to the habit by giving it form.

4. Spots contribute to the habit by giving it points of termination.

5. $\varphi$ is true in $M$ iff there exists a habit of action associated with $\varphi$ by which we can choose suitable courses of actions and seek and find suitable individuals from our universes of discourse, and the constituents of $\varphi$ contribute to the habit by giving it form or points of termination.
The Proof, in GTS

1. A graph $\varphi$ is true in $M$ iff there exists a winning strategy for the Graphist in a semantic game $G(\varphi, M)$.

2. The Graphist has a winning strategy in $G(\varphi, M)$ iff there exists a habit of action associated with $\varphi$ by which we can choose suitable courses of actions and seek and find suitable elements from our universes of discourse.

3. Juxtaposition, polarities of areas and continuous connections between subspaces contribute to the habit by giving it form.

4. Spots contribute to the habit by giving it points of termination.

5. $\varphi$ is true in $M$ iff there exists a habit of action associated with $\varphi$ by which we can choose suitable courses of actions and seek and find suitable individuals from our universes of discourse, and the constituents of $\varphi$ contribute to the habit by giving it form or points of termination.
The Proof, in GTS

1. A graph $\varphi$ is true in $M$ iff there exists a winning strategy for the Graphist in a semantic game $G(\varphi, M)$.

2. The Graphist has a winning strategy in $G(\varphi, M)$ iff there exists a habit of action associated with $\varphi$ by which we can choose suitable courses of actions and seek and find suitable elements from our universes of discourse.

3. Juxtaposition, polarities of areas and continuous connections between subspaces contribute to the habit by giving it form.

4. Spots contribute to the habit by giving it points of termination.

5. $\varphi$ is true in $M$ iff there exists a habit of action associated with $\varphi$ by which we can choose suitable courses of actions and seek and find suitable individuals from our universes of discourse, and the constituents of $\varphi$ contribute to the habit by giving it form or points of termination.
The Proof, in GTS

1. A graph $\varphi$ is true in $M$ iff there exists a winning strategy for the Graphist in a semantic game $G(\varphi, M)$.

2. The Graphist has a winning strategy in $G(\varphi, M)$ iff there exists a habit of action associated with $\varphi$ by which we can choose suitable courses of actions and seek and find suitable elements from our universes of discourse.

3. Juxtaposition, polarities of areas and continuous connections between subspaces contribute to the habit by giving it form.

4. Spots contribute to the habit by giving it points of termination.

5. $\varphi$ is true in $M$ iff there exists a habit of action associated with $\varphi$ by which we can choose suitable courses of actions and seek and find suitable individuals from our universes of discourse, and the constituents of $\varphi$ contribute to the habit by giving it form or points of termination.
The Proof, in GTS

1. A graph $\varphi$ is true in $M$ iff there exists a winning strategy for the Graphist in a semantic game $G(\varphi, M)$.

2. The Graphist has a winning strategy in $G(\varphi, M)$ iff there exists a habit of action associated with $\varphi$ by which we can choose suitable courses of actions and seek and find suitable elements from our universes of discourse.

3. Juxtaposition, polarities of areas and continuous connections between subspaces contribute to the habit by giving it form.

4. Spots contribute to the habit by giving it points of termination.

5. $\varphi$ is true in $M$ iff there exists a habit of action associated with $\varphi$ by which we can choose suitable courses of actions and seek and find suitable individuals from our universes of discourse, and the constituents of $\varphi$ contribute to the habit by giving it form or points of termination.
The Proof, in GTS

6. The constituents of \( \varphi \) contribute to the truth-conditions of \( \varphi \) by giving form or points of termination to some habit associated with \( \varphi \) by which we can choose suitable courses of actions... and \( \varphi \) has truth-conditions only if there exists a habit for \( \varphi \) by which we can choose suitable courses of actions...

7. If the truth-conditions for \( \varphi \) constitute its meaning, the constituents of \( \varphi \) are meaningful by giving form or points of termination to the habit associated with \( \varphi \) and \( \varphi \) is meaningful by being associated with the habit by which we can choose suitable courses of actions...

8. \( \rightarrow \) Constituents of \( \varphi \) are meaningful by giving form or points of termination to the habit associated with \( \varphi \) and \( \varphi \) is meaningful by being associated with the habit by which we can choose suitable courses of actions and seek and find suitable individuals from our universes of discourse.
The Proof, in GTS

6. The constituents of \( \varphi \) contribute to the truth-conditions of \( \varphi \) by giving form or points of termination to some habit associated with \( \varphi \) by which we can choose suitable courses of actions. . . , and \( \varphi \) has truth-conditions only if there exists a habit for \( \varphi \) by which we can choose suitable courses of actions. . .

7. If the truth-conditions for \( \varphi \) constitute its meaning, the constituents of \( \varphi \) are meaningful by giving form or points of termination to the habit associated with \( \varphi \) and \( \varphi \) is meaningful by being associated with the habit by which we can choose suitable courses of actions. . .

8. \( \rightarrow \) Constituents of \( \varphi \) are meaningful by giving form or points of termination to the habit associated with \( \varphi \) and \( \varphi \) is meaningful by being associated with the habit by which we can choose suitable courses of actions and seek and find suitable individuals from our universes of discourse.
The Proof, in GTS

6. The constituents of \( \varphi \) contribute to the truth-conditions of \( \varphi \) by giving form or points of termination to some habit associated with \( \varphi \) by which we can choose suitable courses of actions... and \( \varphi \) has truth-conditions only if there exists a habit for \( \varphi \) by which we can choose suitable courses of actions...

7. If the truth-conditions for \( \varphi \) constitute its meaning, the constituents of \( \varphi \) are meaningful by giving form or points of termination to the habit associated with \( \varphi \) and \( \varphi \) is meaningful by being associated with the habit by which we can choose suitable courses of actions...

8. \( \rightarrow \) Constituents of \( \varphi \) are meaningful by giving form or points of termination to the habit associated with \( \varphi \) and \( \varphi \) is meaningful by being associated with the habit by which we can choose suitable courses of actions and seek and find suitable individuals from our universes of discourse.
Conclusions

We may spell out the logical structure of pragmatic meaning in terms of the relational structure of interaction (=the extensive form of semantic games) that the application of habits gives rise to. In other words, interaction between Graphist and Grapheus (agent–environment) produces a geometry which is the meaning of the assertion.

➤ The relational structure depicts all the ‘conceivable effects’ (payoff distributions at terminal histories) that an intellectual concept may possibly have.

➤ The existence of habits (winning/stable strategies) agrees with the truth of assertions.

The most perfect account of a concept that words can convey will consist in a description of that habit which [the concept] is calculated to produce. But how else can a habit be described than by a description of the kind of action to which it gives rise? (MS 318, 1907)
Conclusions

We may spell out the logical structure of pragmatic meaning in terms of the relational structure of interaction (=the extensive form of semantic games) that the application of habits gives rise to. In other words, interaction between Graphist and Grapheus (agent–environment) produces a geometry which is the meaning of the assertion.

- The relational structure depicts all the ‘conceivable effects’ (payoff distributions at terminal histories) that an intellectual concept may possibly have.
- The existence of habits (winning/stable strategies) agrees with the truth of assertions.

*The most perfect account of a concept that words can convey will consist in a description of that habit which [the concept] is calculated to produce. But how else can a habit be described than by a description of the kind of action to which it gives rise? (MS 318, 1907)*
Conclusions

Three dimensions are necessary and sufficient for the expression of all assertions; so that, if man’s reason was originally limited to the line of speech (which I do not affirm), it has now outgrown the limitation. (MS 654: 6, 1910, Preface to Essays on Meaning).

A year later:

At great pains, I learned to think in diagrams, which is a much superior method [to symbols]. I am convinced that there is a far better one, capable of wonders; but the great cost of the apparatus forbids my learning it. It consists in thinking in stereoscopic moving pictures. (MS L 231, 1911, Letter to Kehler)

(See Pietarinen 2005a for some suggestions.)
Literature

- -, 2005a. Signs of Logic: Peircean Themes on the Philosophy of Language, Games, and Communication (Synthese Library 329), Dordrecht: Springer.
- -, 2005c. Peirce’s contributions to possible-worlds semantics, Studia Logica 79.
http://web.clas.ufl.edu/users/jzeman/