P201 Notes on oscillations and waves

Harmonic motion (oscillation) described by the function

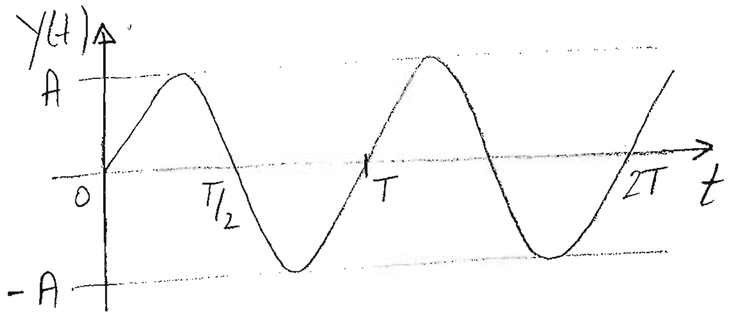
$\boxed{y(t) = A \sin(\omega t + \varphi_0)}$ need to know A, ω, φ_0

A = amplitude of oscillation

ω = angular frequency

φ_0 = initial phase : $y(t=0) = A \sin \varphi_0$

Ex: Plot y vs. t for $\varphi_0 = 0$:



Note: The oscillator moves up & down not to the right!

Ex: Plot y vs. t for $\varphi_0 = \frac{\pi}{2}$.

Ex: Plot y vs. t for $\varphi_0 = \frac{\pi}{4}$

Superposition (addition) of oscillations:

$Y_{net}(t) = Y_1(t) + Y_2(t)$

where $\quad Y_1(t) = A_1 \sin(\omega_1 t + \varphi_{01})$
$\quad Y_2(t) = A_2 \sin(\omega_2 t + \varphi_{02})$

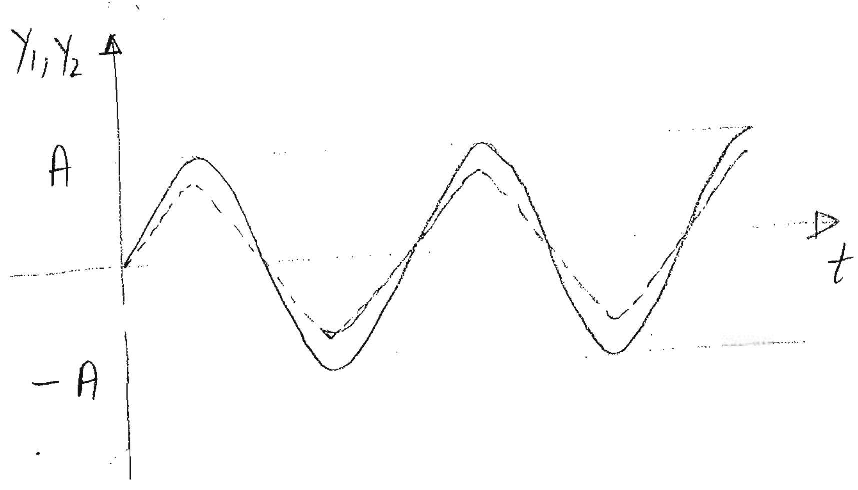
For a simpler case, take $\omega_1 = \omega_2 = \omega$ and $\varphi_{01} = 0$.

Then $Y_{net}(t) = A_1 \sin(\omega t) + A_2 \sin(\omega t + \varphi_{02})$

* Note: φ_{02} is the phase difference between Y_1 & Y_2.

Examples

Take $A_2 = 0.8 A_1$ and $\varphi_{02} = 0$ (in-phase)



Y_1 solid line
Y_2 dashed line

Show $Y_1(t) + Y_2(t)$ on the graph!

Now take $\varphi_{02} = \pi$ and show y_1, y_2 and y_{net}.

Obs. * if the phase difference is zero, the addition of y_1 and y_2 produce a net oscillation with a larger amplitude.

$$A_1 \sin \omega t + A_2 \sin \omega t = (A_1 + A_2) \sin \omega t = 1.8\, A_1 \sin \omega t$$

* if the phase difference is π, the net amplitude is smaller than both A_1 & A_2:

$$A_1 \sin \omega t + A_2 \sin(\omega t + \pi) = A_1 \sin \omega t - A_2 \sin \omega t$$
$$= (A_1 - A_2) \sin \omega t = 0.2\, A_1 \sin \omega t$$

Note that if $A_1 = A_2$, the net oscillation has a zero amplitude, i.e the oscillator does not oscillate!

Obs When an oscillator interacts with nearby objects (materials), the nearby objects can be set in oscillation; i.e., energy is transferred from one oscillator to the next.

How? Through work done by forces between oscillators, e.g., segments of rope, string, etc.

Definition: The propagation (transfer) of perturbation through materials is called a mechanical wave.

A wave carries energy from a primary oscillator, called wave source, to remote oscillators.

Q: How fast??

A: It depends on the material.

e.g. string or rope: $v = \sqrt{\frac{TL}{M}}$ T = tension, L = rope length, M = rope mass

sound waves in fluids → $v = \sqrt{\frac{B}{\rho}}$ B = fluid bulk modulus, ρ = fluid density

in solids (e.g. rod) → $v = \sqrt{\frac{Y}{\rho}}$ Y = Young's modulus

Obs.: energy transfer takes time $\Rightarrow$ neighboring oscillators will be delayed with respect to the source.

$\Rightarrow$ neighboring oscillators are not in phase.

$\rightarrow$ this delay determines the speed of the wave.

e.g. a tighter string carries faster waves

a denser material carries sound faster

(due to stronger restoring forces)

$\Rightarrow$ an oscillator at distance d away from the source will oscillate according to

$$y(t,d) = A \sin[\omega(t-\tau)]$$

means y as a function of t and d.

and τ is the time delay

$$\tau = \frac{d}{v}$$, where v is the speed of the wave.

We can write

$$y(t,d) = A \sin\left(\omega t - \omega \frac{d}{v}\right)$$

and using $\omega T = 2\pi$:

$$y(t,d) = A \sin\left[2\pi\left(\frac{t}{T} - \frac{d}{v\,T}\right)\right]$$

$vT \stackrel{\text{def}}{=}$ wavelength λ = the distance the wave travels in one period of oscillation.

So then

$$\boxed{y(t,d) = A \sin\left[2\pi\left(\frac{t}{T} - \frac{d}{\lambda}\right)\right]}$$

T & λ are natural units of time and distance

Waves are also written as

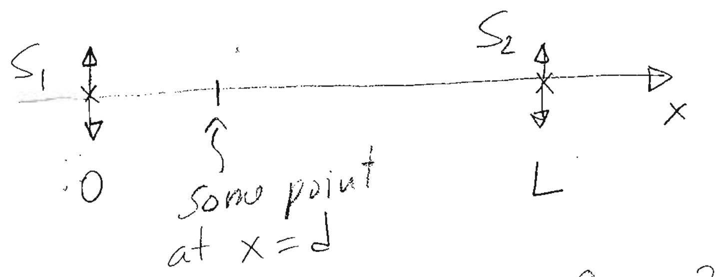
$$y(t,d) = A \sin(\omega t - K d)$$

where $K = \frac{2\pi}{\lambda}$ is called the wave number.

Wave interference (superposition of oscillations)

Consider: two wave sources S_1 & S_2 at distance L from one another, oscillating with the same ω and having the same phase (φ).

The two emitted waves are said to be coherent.



Take $L = 4m$ and $\lambda = 3m$.

Q: At what point(s) between the sources do the waves interfere constructively?

The difference in phase at $x=d$ is

$$\boxed{\Delta\varphi(d) = 2\pi\frac{d}{\lambda} - 2\pi\frac{L-d}{\lambda} = \frac{2\pi}{\lambda}(2d-L)}$$

Constructive interference for $2d - L = 0$

$\Rightarrow d = \frac{L}{2}$ i.e, mid-way as expected.

Q: How about destructive interference?
(cancellation)

A: Need a phase difference of $\pm\pi$.

$$\frac{2\pi}{\lambda}(2d-L)=\pi \Rightarrow d=\frac{L}{2}+\frac{\lambda}{4}=2.75\,m$$

$$\frac{2\pi}{\lambda}(2d-L)=-\pi \Rightarrow d=\frac{L}{2}-\frac{\lambda}{4}=1.25\,m$$

Note: If the waves from S_1 & S_2 have the same amplitude $\Rightarrow$ no oscillation (vibration) between sources at $d=1.25\,m$ & $2.75\,m$

General rule for interference of coherent waves:

$$\Delta\varphi = 2\pi\frac{\Delta l}{\lambda}$$ where Δl = path length

Constructive interference for $\Delta l = \ldots -2\lambda, -\lambda, 0, \lambda, 2\lambda, \ldots$

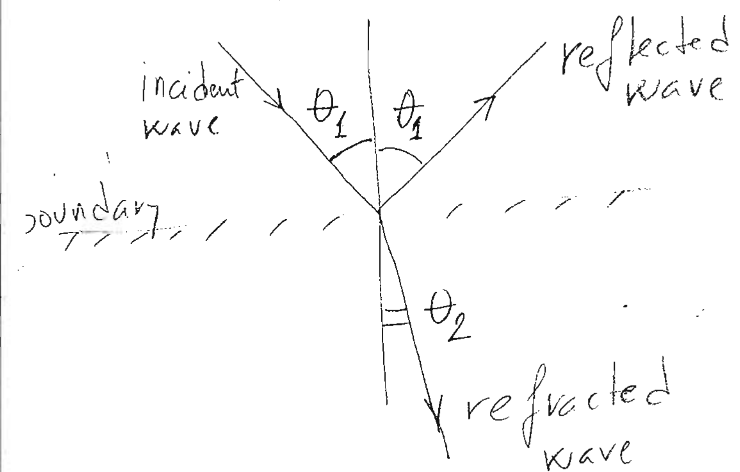
Destructive interference for $\Delta l = \ldots -\frac{3\lambda}{2}, -\frac{\lambda}{2}, \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots$

Reflection and refraction

* Occur at boundaries between materials.
* The frequency of the wave does not change, but other things might.

≡ Things that change during reflection:
- direction of propagation
- phase

≡ Things that change during refraction:
- direction of propagation
- wave speed
- wave length



Law of reflection:
same θ_1 angle

Law of refraction

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

A standing wave can be created by the superposition of two coherent waves traveling in opposite directions, e.g. vibrations in musical instruments (strings & pipes).

Note: in a standing wave, all oscillators are in phase!

Obs: The length of strings and pipes determine the possible wavelength & frequencies of standing waves.

$\Downarrow$

called natural or resonant frequencies

For strings: $n \frac{\lambda}{2} = L$ with $n = 1, 2, 3, \ldots$

For pipes: $n \frac{\lambda}{2} = L$ with $n = 1, 2, 3, \ldots$ both ends open

$n \frac{\lambda}{4} = L$, $n = 1, 3, 5, \ldots$ (odd!) one end closed