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Reconciling Kuznets and Habbakuk in a Unified Growth Theory

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ABSTRACT

Economic historians have debated the relative labor productivity of the United States agricultural sector during the 19th century. David (2005) offers a reconciliation of the opposing views by suggesting that while productivity per hour worked in agriculture was high, the number of hours worked per year was low. We model and extend a version of Davis's reconciliation within a unified growth theory that connections the structural transformation away from traditional agriculture to several other features of United States development. Similar to David, our model predicts an almost two-fold annual worker-productivity advantage in the modern (industrial) sector of the economy, entirely due to greater hours worked per year. The dynamic general equilibrium model is consistent with the structural transformation having minor direct and indirect effects on aggregate labor productivity *per hour*, but substantial effects on aggregate labor productivity *per worker*. The model also provides a reasonable match to the trends in schooling, fertility, rates of return to physical capital, and labor productivity growth over the two centuries.

I. Introduction

This paper studies the transformation of an economy from one based on traditional agriculture to one based on modern, capital-intensive modes of production. We take a unified approach that examines the structural transformation over long periods of time, two centuries of United States growth from 1800 to 2000, and that includes several other features of development such as schooling, fertility, physical capital intensity, and the pattern of economic growth rates over time. We argue that the structural transformation is connected with these other features of development in potentially important ways. Thus, a robust explanation of the causes and consequences of the structural transformation must be consistent with the historical patterns of *all* related variables.

The motivation for our study begins with the debate over the relative productivity of the agricultural sector of the United States during the 19th century. Kuznets (1965) began a tradition of thought based on the idea that agricultural workers were less productive than non-agricultural workers. If true, the structural transformation should have a direct and positive impact on economic growth, as workers shift from relatively low to relatively high productivity employment. Habbakuk (1962) began an opposing tradition based on the idea that labor productivity in agriculture was high in the early 19th century and remained so, despite population growth, due to the abundance of land in the United States. From the perspective of this tradition, high labor productivity in agriculture both slowed the structural transformation and eliminated any direct benefit of the reallocation of labor on economic growth.

David (2005) offers a reconciliation of these two traditions. He argues that worker productivity was high in agriculture on an hourly basis, but that agricultural workers worked far fewer hours over the course of the year due to seasonal constraints on outdoor production. David suggests that the hourly marginal product of labor was similar across sectors. However, the average product of labor in agriculture was actually higher because it included significant land rents that were paid to landed workers as part of their "wage." The land rents compensated landed workers for their shorter work year, so that annual wages were roughly equalized across sectors—consistent with an "equilibrium" flow of labor across sectors.

David's reconciliation is also consistent with the data on aggregate economic growth in the 19th century. If his reconciliation is correct, then the structural transformation has little effect on the aggregate growth of worker productivity *per hour* (since marginal products are roughly equalized on an hourly basis), but has a significant effect on the aggregate growth rate of output *per worker* (because hours of work per year expands). Table 1 reports the annualized growth rate in labor productivity per hour worked and Table 2 reports the annualized growth rate in labor productivity per worker. In the 19th century, growth rates on a per worker basis were much higher than those on a per hour basis, consistent with the reconciliation.

We formally model a version of David's reconciliation within a unified growth theory. We clarifiy David's argument by pointing that land rents can only compensate for a shorter-paid work-year, if the land is passed down from one generation of traditional farmers to another. If land is instead purchased by each successive generation, then land cannot be a source of lifetime income, but rather only serves to *transfer* lifetime income over the life-cycle (as any asset purchase does). Traditional workers will stay tied to the land, despite the shorter work-year, only if they inherit the land from their parents.

We then extend David's analysis by including several variables that are potentially related to the inheritance of land in traditional agriculture: fertility, saving, and schooling. Rural households in United States history had much higher fertility rates than urban households (see, for example, Greenwood and Seshadri (2002)). Carter, Ransom, and Sutch (2002) argue that this could be due to a higher demand for children associated with "family farming." They also suggest that owning a family farm provides income later in life, which reduces the incentive to save for retirement. The structural transformation away from traditional farming should then reduce fertility rates and increase saving rates. Both of these consequences of the structural transformation should increase physical capital intensity. Others have argued that schooling is relatively less productive in traditional agriculture (see, for example, Caselli and Coleman (2001)). The structural transformation may then play a role in explaining the rise in schooling over the course of development. A robust theory of the structural transformation in the United States must explain these other behaviors and assess how they combine to generate the growth rates presented in Tables 1 and 2.

The main findings from our quantitative analysis are as follows.

There were substantial wage/productivity gaps across sectors during United States development due to the relative short-fall in hours worked in traditional agriculture. We estimate the short-fall of work hours in traditional agriculture indirectly by using the land rent-wage ratio to explain the observed gap in fertility across rural and urban households. In equilibrium, the resulting land rent-wage ratio then implies a wage short-fall resulting from the seasonal constraint on hours worked in the traditional sector. In addition, hours worked per worker will be lower in the traditional sector because of greater fertility, which reduces adult labor hours and increases the amount of (part-time) child labor. We estimate that hours worked per worker in the traditional sector was 56 percent of the hours worked per worker in the modern sector.

The structural transformation away from traditional agriculture did not increase human or physical capital intensity per worker during the 19th century.

In our theory, schooling turns out to be unaffected by the inheritance of land and land rents. This implies that schooling is also unaffected by the structural transformation. In addition, despite the fact that fertility is raised and saving is lowered by land ownership, the structural transformation away from traditional farming did not serve to raise physical capital intensity per worker in the 19th century. As workers moved out of traditional farming and into the capital-intensive modern sector, more physical capital was required to maintain physical capital intensity per worker. The increase in required physical capital approximately offset the effects of lower population growth and greater saving per household.

Three quarters of the 19th century gap in labor productivity growth per worker over labor productivity growth per hour worked can be explained by the expansion in hours worked.

The structural transformation did *not* increase growth in labor productivity per hour *directly* (because labor productivity per hour was roughly equalized across sectors) or *indirectly* (because it did not raise physical or human capital intensity). However, it did raise the growth rate in labor productivity per worker because of the expansion in hours worked per worker.

The majority of the growth in labor productivity per hour from 1800 to 2000 was due to factor accumulation.

We estimate that about 60 percent of the growth in productivity per hour worked was due to increases in schooling, physical capital intensity, and the adult-equivalent labor supply per worker (as the fraction of child labor declined).

II. Related Literature

The general equilibrium analysis of long periods of growth, covering both traditional and modern production regimes, began with Galor and Weil (1996, 2000) and has expanded rapidly in the last ten years. Galor (2005) presents a survey of this literature that he terms "unified theories of growth." One feature of developing economies that has not been fully integrated into a unified theory of growth is the presence of wage gaps across sectors. Some recent papers have introduced wage gaps based on differences in education requirements across sectors (Doepke (2004) and Greenwood and Seshadri (2002)). However, as suggested by observations from the 19th century United States, there are significant wage gaps across sectors for workers with similar levels of education. To our knowledge no one has previously attempted to explain wage gaps, for given levels of education, in a two-sector unified growth theory that also includes schooling, fertility, and the structural transformation.

Economic historians have studied wage gaps in the United States and other currently developed countries. The main finding in this literature is that the gaps in wages per hour across industry and agriculture were relatively small. Margo (2000) estimates very small wage gaps for the United States in the 19th century. Hatten and Williamson (1992) find relatively small wage gaps in the United States from 1890 to

1920, ranging from 5 to 30 percent. They find larger gaps from 1920 to 1940 which are mostly explained by the high urban unemployment rates during the Great Depression. Hatton and Williamson (1991, Figure 1) present data that demonstrates wage gaps of less than 20 percent in the United Kingdom, Germany, and Sweden from 1855 to 1915. Sicsic (1992) finds wage gaps of 0 to 25 percent in France from 1852 to 1892. The general finding of small wage gaps per hour motivated our decision to focus on differences in hours worked as the cause of annual differences in wages and productivity across sectors.

A separate literature has studied the wage/productivity gaps in currently developing countries, where the measured gaps are much larger than in history (Gollin et al (2004)). The explanations offered for wage/productivity gaps include compensation for urban unemployment (Harris and Todaro (1970)), education gaps (Cordoba and Ripoll (2006)), unmeasured home production (Gollin et al (2004)), and migration costs (Restuccia et al (2004)). Our theory complements these explanations by emphasizing differences in hours worked due to differences in seasonal constraints on hours, child rearing, and child labor.

III. Model

This section extends the one-sector neoclassical growth model of Mourmouras and Rangazas (2007) to two sectors by including a traditional and a modern sector similar to Hansen and Prescott (2002). As in Hansen and Prescott, the traditional sector differs from the modern sector because land is an input in traditional production. Consistent with the fact that labor shares show no systematic trend over the course of development, the two sectors have equal labor shares. Thus, the physical capital share, excluding land, is lower in the traditional sector.

The model differs from Hansen and Prescott in several ways. The differences in modeling enable us to capture and quantify David's reconciliation of the traditions began by Kuznets and Habbakuk. The differences also allow us to establish connections between traditional farming and other features of development that may affect the growth of labor productivity.

First, Hansen and Prescott assume that there is a perfectly competitive market for land. They also assume that land must be purchased by each successive generation—i.e. there is no bequest motive that causes land to be passed down. We assume that land is passed from one generation of landowners to the next either because of the absence of a land market or because of a preference of landowners to keep the land in the family. Our approach is similar to Drazen and Eckstein (1988), who assume that in the absence of formal land markets, farm land itself, is passed from one generation of farmers to the next. Our approach differs from theirs because we assume the land is passed *only if* the children agree to work in the traditional sector themselves—this ties the inheritance/management of land to the occupational choice of workers.[1] This assumption creates two household types—landed households that only work in the traditional sector and landless households that work at least some fraction of the time in the modern sector.

Second, we assume that the traditional technology can only be operated for a fraction f of each period. The motivation for the assumption is that production depending on land inputs is constrained by the weather and the length of day, both of which vary with the seasons. Thus, landed households can only supply their labor for the same fraction *f* of the period. The shorter production period, however, does not generate extra leisure. Instead during the off-period, landed workers spend unpaid time securing, maintaining, and improving the land and traditional technology that they will inherit when old.[2] In equilibrium, the constraint on the paid labor supply of landed traditional workers is compensated by the land rents they receive from inheriting the traditional technology.

Finally, the quality (human capital) and quantity (fertility) of children are endogenous household choices. This enables us to connect traditional production to fertility and schooling as suggested in other research (e.g. Carter et al (2002), Doepke (2004), and Lord and Rangazas (2006)).

Our assumptions generate an equilibrium interpretation for the allocation of labor across the two sectors that is similar to the one implied by Davis's reconciliation of Kuznets and Habbakuk. The annual productivity and wages of traditional agricultural workers are relatively low as Kuznets argues. However, this is due to a shorter work year for those who are tied to the traditional sector. As Habbakuk argued, traditional farm labor is as productive, and as well-paid per hour, on an hourly basis as is labor in the modern sector. In the presence of high fertility, improvements and expansion of productive land helps maintain productivity and allows some fraction of each generation to remain in traditional agriculture. The constraint on the length of the paid work-year in the traditional sector is compensated for by entrepreneurial income/land rents when the traditional technology/land is inherited later in life. Labor reallocation each period equalizes lifetime income across sectors.

A. *Technology*

1. Modern Sector

Producers in the modern sector are neoclassical firms with the standard Cobb-Douglas production function

(1) $$Y_t = A_t K_t^{\alpha} \left(H_t\right)^{1-\alpha},$$

where Y denotes output, K is the physical capital stock, H is the human capital input, and α is the capital share parameter. The level and growth of A is exogenously determined, with a constant growth rate of g.

Modern sector firms operate in perfectly competitive markets. The standard profit-maximizing factor-price equations for the rental rates on human and physical capital are

(2a) $$W_t = \left(1-\alpha\right) A_t^{\frac{1}{1-\alpha}} k_t^{\alpha}$$

(2b) $$r_t = \alpha k_t^{\alpha-1},$$

where r is rental rate paid to physical capital and $k \equiv K / HA^{\frac{1}{1-\alpha}}$.

2. Traditional Sector

The traditional sector's technology is directly operated by a traditional household. On a per household basis, the technology is given by

(3) $$O_t = f\tilde{A}_t l_t^{\rho} \tilde{K}_t^{\phi} \tilde{H}_t^{1-\phi-\rho},$$

where ρ and ϕ are the constant land and capital shares. As in Hansen and Prescott (2002), traditional output, O, is a perfect substitute for the output of the modern sector. Due to seasonal variation in weather conditions, the traditional technology can only be operated a fraction, f, of the period. Traditional TFP, $\tilde{A}$, will in general grow at different exogenous rates than the modern sector TFP, *A*. The difference in growth rates may be due, in part, to the fact that the growth in $\tilde{A}$ is interpreted to include improvements and expansion of productive land. The stock of raw land per household, *l*, is the aggregate stock of raw land (assumed to be fixed) divided by the number of households in the traditional sector. The physical and human capital stocks used by a traditional sector household are given by $\tilde{K}$ and $\tilde{H}$.[3]

The land input is inherited from parents, so the only choices are how much physical and human capital to use. The traditional sector household operates in the same competitive factor markets as modern firms and must pay the same factor prices. The conditions for maximizing entrepreneurial income/land rents by employing human and physical capital are given by

(4a) $$W_t = (1-\phi-\rho) A_t^{\frac{1}{1-\alpha}} f\tilde{a}_t l_t^{\rho} \tilde{k}_t^{\phi} \left(\tilde{H}_t A_t^{\frac{1}{1-\alpha}} \right)^{-\rho}$$

(4b) $$r_t = \phi f\tilde{a}_t l_t^{\rho} \tilde{k}_t^{\phi-1} \left(\tilde{H}_t A_t^{\frac{1}{1-\alpha}} \right)^{-\rho},$$

where $\tilde{a} \equiv \tilde{A}/A$ and $\tilde{k} \equiv \tilde{K}/\tilde{H}A^{\frac{1}{1-\alpha}}$.

B. Households

There are two types of households, landed and landless. The landed households are households that inherit the traditional technology/land from their parents. Parents are only willing to pass the land on to those children who remain in the traditional sector fulltime. Thus, the paid labor supply of landed households is constrained by the length of the work-year in the traditional sector. Landless households are free to work in either sector.

1. Landless Households

All households live for three periods; one period of childhood and two periods of adulthood. Households value their consumption over the two periods of adulthood (c_t^y, c_{t+1}^o), the number of children (n_{t+1}), and the adult human capital of children(h_{t+1}). Preferences are identical across households, regardless of occupational choice, and are given by

(5) $$U_t = \ln c_t^y + \beta \ln c_{t+1}^o + \psi \ln n_{t+1} + \varepsilon \ln h_{t+1},$$

where 0 < β < 1, $\varepsilon > 0$, and ψ > 0 are preference parameters. This preference specification is a simple way of capturing the idea that parents value both the quantity and the quality of children. It has been use extensively in the literature on fertility and growth (e.g. Galor and Weil (2000), Greenwood and Seshadri (2002), Hazan and Berdugo (2002), and Moav (2005)).

Landless adults inelastically supply one unit of labor when young and zero units when old. Children have an endowment of $T < 1$ units of time that they can use to attend school (e_t) or work ($T - e_t$). Children have less than one unit of time to spend productively because in the very beginning years of childhood they are too young to either attend school or to work, and in their middle years they do not have the mental or physical endurance to school or work as long as an adult.

While children may work as they become older, they are also expensive to care for and feed. To raise each child requires a loss of adult work time, resulting in a loss of consumption equal to a fixed fraction τ of the adult's first period wages.

Children are too young to work during their very early years. So each child invests least $\bar{e}$ units of time into learning during the first portion of their childhood. This gives older children $\gamma \bar{h}_t = \gamma \bar{e}^{\theta}$ units of human capital that can be used in production during the later years of childhood, where 0 < θ <1 is a parameter that gauges the effect of schooling on human capital accumulation and $0 < \gamma < 1$ reflects the fact that children lack relative physical strength or experience in applying knowledge to production compared to an adult. Adult human capital of the same person in the next period is $h_{t+1} = e_t^{\theta}$. Thus, a person is more productive in adulthood than in childhood because of greater strength and experience (1 > γ) and additional schooling ($e_t \geq \bar{e}$).

The landless household maximizes utility subject to the lifetime budget constraint,

$$c_t^y + \frac{c_{t+1}^o}{R_t} + n_{t+1}\tau w_t h_t = w_t h_t + n_{t+1} w_t \gamma \bar{h}(T - e_t),$$

where the return on the ownership of physical capital, purchased in period *t* and rented to producers in period *t+1*, is $R_t \equiv r_{t+1} + 1 - \delta$, where δ is the rate of depreciation of physical capital. To simplify some of the expressions, we assume δ = 1, so that $R = r$.

In addition to the standard first order conditions for life-cycle consumption, the choices of n_{t+1} and e_t yield

(6a) $$\frac{\varepsilon\theta}{e_t} = \lambda_t n_{t+1} w_t \gamma \bar{h}$$

(6b) $$\frac{\psi}{n_{t+1}} = \lambda_t \left[\tau w_t h_t - (T - e_t) w_t \gamma \bar{h}\right],$$

where λ_t is the Lagrange multiplier.

Equation (6) summarizes the parents' decisions regarding the quantity and quality of children. Equation (6a) says the marginal utility of additional child quality must be equated to the marginal value of consumption lost from allowing children of working age to attend school. Equation (6b) says the marginal utility of additional children must be equated to the marginal value of lost consumption. Consumption is lost from having additional children because we assume the cost of children exceeds the earnings that older children bring to the household.

Solving the model gives us the following demand functions for children, schooling, and savings

(7a) $$n_{t+1} = \frac{\psi}{(1+\beta+\psi)\left(\tau - \gamma(T - e_t)(\bar{e}/e_{t-1})^{\theta}\right)}$$

(7b) $$e_t = \frac{\varepsilon\theta\left(\tau(e_{t-1}/\bar{e})^{\theta} - \gamma T\right)}{\gamma(1-\varepsilon\theta)}$$

(7c) $$s_t = \left[\frac{\beta}{1+\beta+\psi}\right] W_t h_t$$

Assuming that e_{t-1} is sufficiently high, an assumption that we make throughout, a dynamic results that causes economic growth and a demographic transition.[4] Greater schooling raises adult earnings relative to older children's earnings. This raises the net cost of having children, so fertility declines. Lower fertility and greater consumption lowers both the quantity and the value of forgone earnings from schooling children, so schooling rises further. Thus, the sole factor driving fertility down is the rise in schooling.

2. *Landed Households*

In the case of landed households, we assume that the skills and land needed to operate a traditional technology are inherited from parents, but only if children tie their labor to the traditional sector when young. Operating the traditional technology will generate residual income, or land rents, for the household during the second period of adulthood that compensates for accepting the shorter paid work-years when young.

The lifetime budget constraint for households choosing to follow their parents and work in the traditional sector is

(8) $$\tilde{c}_t^y + \frac{\tilde{c}_{t+1}^o}{R_t} + \tilde{n}_{t+1}\tau W_t f\tilde{h}_t = W_t f\tilde{h}_t + \tilde{n}_{t+1}W_t\gamma f\bar{h}\left(T - \tilde{e}_t\right) + \frac{O_{t+1} - W_{t+1}\tilde{H}_t - r_{t+1}\tilde{K}_{t+1}}{R_t},$$

where we assume that the land is passed on at death to the next generation of traditional workers from the family. Assuming that the residual income accrues in the third period captures the idea that the family farm provided a source of retirement income that substitutes for retirement saving, a potentially important factor in slowing physical capital accumulation in the early stages of development (see Carter et al (2002)).

Maximizing utility subject to (5) and (8) yields the standard life-cycle Euler equation for consumption, the first order conditions for maximizing residual income (given by (4)), and the following demands for children, schooling, and savings

(9a) $$\tilde{n}_{t+1} = \left(\frac{\psi}{1+\beta+\psi}\right)\frac{\left(1 + \frac{\rho O_{t+1}}{R_t W_t \tilde{h}_t f}\right)}{\tau - \gamma\left(T - \tilde{e}_t\right)\left(\bar{e}/\tilde{e}_{t-1}\right)^{\theta}}$$

(9b) $$\tilde{e}_t = \frac{\varepsilon\theta\left(\tau\left(\tilde{e}_{t-1}/\bar{e}\right) - \gamma T\right)}{\gamma\left(1 - \varepsilon\theta\right)}$$

(9c) $$s_t = \left[\frac{\beta}{1+\beta+\psi} - \frac{1+\psi}{1+\beta+\psi} \frac{\rho O_{t+1}}{R_t W_t f \tilde{h}_t} \right] W_t f \tilde{h}_t .$$

Contrasting (9a) to (7a), reveals that operating the traditional technology introduces a new term in the fertility demand function, $\rho O_{t+1} / R_t W_t \tilde{h}_t f$, that raises fertility. The numerator of this term is the share of family production that flows to older households. The denominator is the potential "full" wage that can be earned as a young traditional sector worker, which determines the opportunity cost of having children. The more important family production is, relative to the opportunity cost of children, the stronger is the demand for children. This is not a pure wealth effect, but rather is an effect that arises when one form of wealth (that does *not* affect the net cost of children), the ownership of a traditional production technology, changes relative to another form of wealth (that *does* affect the net cost of children), the flow of adult earnings. A shift in the composition of family wealth away from family production and toward adult labor income causes the net cost of children to rise, for a given level of family wealth, and the demand of children falls.

The demand for schooling in (9b) takes the same form as in (7b), although there may be different initial conditions for households operating the traditional sector. We do not explore this possibility here, and assume throughout that the initial conditions for schooling are the same for both types of families. As a result, the path of schooling will be the same for both types of households.[5] So, we can simplify notation by writing $\tilde{e}_t = e_t$ and $\tilde{h}_t = h_t$.

There is no effect of residual income on schooling because of two offsetting effects. To see these effects, first note that fertility raises the cost of schooling children (more children means greater forgone consumption of parents as schooling rises and child labor income falls). Second, note that the level of parental consumption determines the marginal *value* of forgone consumption associated with greater schooling (higher parental consumption levels means parents can better "afford" the lost consumption associated with more schooling). Family production raises *both* fertility and parental consumption, other things constant. As just mentioned, higher fertility lowers the incentive to school children, but a higher consumption level raises the incentive to school children. With our functional forms for preferences and human capital production, these two effects always exactly offset. The neutral effect of residual income on schooling implies that the structural transformation per se will *not* increase schooling. This turns out to be important when the model is applied to explain United States growth in the 19th century.

The savings function in (9c) exhibits the effect of family farming suggested by Carter et al (2002). The presence of second period income from operating the traditional technology discourages retirement saving. The fraction of current period wages that are saved is smaller as a result. So saving per household is lower in the traditional sector for two reasons: wages are lower because of the shorter paid work-year and there is a smaller fraction of earnings that is saved.

3. Effective labor supply

Given our focus, it is worth thinking a bit more about the effective labor supply coming from each type of household. A landed adult will clearly supply less paid labor hours than a landless adult because $f < 1$. However, the number of adult-equivalent labor hours will also differ across households due to differences in adult childcare and in the labor hours supplied by children. In other words, even if traditional households were unconstrained (f = 1) their labor supply would still differ from that of landless households.

The *unconstrained* adult-equivalent labor supply, scaled for the relatively lower productivity of children, of the two household types are defined as $\tilde{m}_t = 1 - \tau \tilde{n}_t + \gamma \frac{\bar{h}}{h_t}\left(T - e_t\right)\tilde{n}_t$ and $m_t = 1 - \tau n_t + \gamma \frac{\bar{h}}{h_t}\left(T - e_t\right) n_t$. The following proposition shows that adult-equivalent household labor supply of landed households is greater than for landed households (proofs of all propositions are found in the Appendix).

Proposition 1 (Adult Equivalent Labor supply) *The unconstrained adult-equivalent labor supply is higher in the modern sector,*

$$m_t = \frac{1+\beta}{1+\beta+\psi} \equiv m > \tilde{m}_t = \frac{1+\beta}{1+\beta+\psi} - \frac{\psi}{1+\beta+\psi}\left(1 + \frac{\rho O_{t+1}}{R_t W_t f h_t}\right).$$

The intuition for *Proposition 1* is that, with the logarithmic utility, total net expenditures on children are constant. Net expenditures are measured in the forgone earnings associated with child rearing net of the earnings of children. Since these net expenditures remain constant, the family's labor supply also remains constant as fertility varies. The unconstrained labor supply of traditional households is lower because they have more children for a *given* net cost of raising children (due to the presence of non-labor income).

C. Equilibrium

We now determine equilibrium in the factor markets for human and physical capital. Given k_t and l_t, the market clearing values of the factor prices and the human and physical capital intensities chosen by a traditional sector producer are given by (2a), (2b), (4a), and (4b). Thus, only the equilibrium values of k_t and l_t remain to be determined.

1. Equilibrium l_t

Determining the equilibrium quantity of raw land per traditional producer is equivalent to finding the number of equilibrium traditional sector producers (since raw land is fixed). For any household to remain in the traditional sector and inherit land from their parents, the lifetime utility of doing so must be at least as great as the utility associated with forgoing the inheritance and becoming unrestricted in the choice of occupation. We assume throughout that land is sufficiently scarce that some children born into a traditional sector household will choose to work in the modern sector. In other words, if all children born in the traditional sector remained in the traditional sector as adults, the quantity of land per producer would be too small to generate enough entrepreneurial income/land rents to compensate for the constraint on paid work, and the lifetime utility of traditional sector households would fall below that of landless households. Thus, the "competition" for land among traditional sector siblings will be strong enough to create an equilibrium exit from the traditional sector that equates the lifetime utility of the landed and landless households in each generation.

The fact that the lifetime utility of the landed and landless households must be equated, gives an equilibrium condition for determining, along with (2) and (4), the number of traditional sector producers in each period.

Proposition 2 (Equilibrium Land Rents) *In equilibrium, land rents satisfy*

(10) $$\frac{\rho O_{t+1}}{R_t W_t f\tilde{h}_t} = \Omega,$$

where $\Omega \equiv \left(\frac{1}{f}\right)^{\frac{1+\beta}{1+\beta+\psi}} - 1.$

The proposition states that the ratio of the present value of land rents to potential wages in the traditional sector is constant in equilibrium. The size of the ratio is determined by the length of the paid work-year in the traditional sector. The greater is the length of the work-year, the lower is the land rent ratio. Land rents per household vary inversely with the number of traditional sector households. Thus, the greater is the length of the paid work-year, the greater are the number of household producing in the traditional sector. Note, *Proposition 2* implies $\tilde{m}_t = \frac{1+\beta-\psi\Omega}{1+\beta+\psi} \equiv \tilde{m}$.

An additional condition is needed to ensure that the traditional sector is not constrained in the labor its uses, i.e. the demand for labor from the traditional sector must be greater than or equal to the supply of labor coming from landed households. If this were not the case, then there would be a "dual" labor market that would give rise to different wages per hour across sectors and across household types. Some hiring of part-time labor from landless households, "farm hands," is necessary to integrate the labor markets.

Using (2) and (4), this additional condition can be written as

(11) $$\tilde{N}_t \tilde{H}_{t+1} = \frac{\left(f\tilde{a}_{t+1}\right)^{\frac{1}{\rho}} \left(\frac{\phi}{\alpha}\right)^{\frac{\phi}{\rho}} L}{k_{t+1} A_{t+1}^{\frac{1}{1-\alpha}}} \geq \tilde{N}_{t+1} f h_{t+1} \tilde{m}.$$

The next proposition gives a convenient expression that guarantees (11) holds by bounding the growth in the relative TFP of the traditional sector.

Proposition 3 (Integrated Labor Markets) *The following condition guarantees that labor markets are integrated, i.e. guarantees that (11) is satisfied in each period*

(12) $$\left(\frac{(1-\alpha)\Omega}{\rho\tilde{m}}r_t\right)^{\rho} \geq \frac{\tilde{a}_{t+1}}{\tilde{a}_t}$$

Proposition 3 is used in our empirical application to verify that (11) holds over the two centuries that we simulate United States growth.

2. Equilibrium k_t

The physical capital intensity is determined by the equilibrium condition that equates the physical capital demanded for production by the two sectors to the savings of the households one period before.

(13) $$K_{t+1} + \tilde{K}_{t+1} = \tilde{N}_t\tilde{s}_t + N_t s_t .$$

The capital market equilibrium condition leads to the following transition equation for physical capital intensity in the modern sector.

Proposition 4 *The physical capital intensity in the modern sector evolves according to the following first order difference equation*

(14) $$k_{t+1} = \left[\frac{\tilde{\pi}_t\tilde{\beta}f + (1-\tilde{\pi}_t)\beta}{1+\beta+\psi} + \frac{\tilde{\pi}_t f\Omega(\alpha-\phi)}{\rho(1+g)}\right]\frac{(1-\alpha)k_t^{\alpha}h_t}{\overline{n}_t h_{t+1}\overline{m}_{t+1}(1+g)^{1/(1-\alpha)}} ,$$

where $\tilde{\pi}_t$ *is the fraction of young landed households at time t,* $\overline{n}_t \equiv \tilde{\pi}_t\tilde{n}_t + \left(1-\tilde{\pi}_t\right)n_t$ *is the average fertility across both household types, the average adult-equivalent labor supply per household is*

$\bar{m}_{t+1} \equiv \tilde{\pi}_{t+1}\tilde{m} + (1-\tilde{\pi}_{t+1})m$, *and the saving rate of a landed household is* $\tilde{\beta} \equiv \beta - (1+\psi)\Omega$.

We can now discuss the avenues through which the structural transformation from traditional to modern production affects physical capital intensity. First, the movement of households out of the traditional sector increases saving because it increases wage income and because it increases the rate of saving ($\tilde{\beta} < \beta$). Another positive effect on physical capital intensity results from the reduced fertility and population growth associated with the transformation, captured by the decline in $\bar{n}_t$. This effect allows a given flow of saving to be spread over fewer workers in the next period.

The positive effects are countered by two negative effects of the transformation on the physical capital intensity. First, the modern sector is relatively more capital intensive ($\alpha > \phi$). As labor moves into the modern sector, the capital *required* to maintain a given capital intensity in the economy rises. The difference in relative capital intensities, and its effect on capital requirements economy-wide, is given by the second term in the square-bracket of (14). As $\tilde{\pi}$ falls, this term become smaller and capital intensity falls for a given flow of aggregate saving. Second, the adult-equivalent labor supply rises during the economic transformation because the adult-equivalent labor supply is higher in the modern sector ($m > \tilde{m}$). So, $\bar{m}$ rises as $\tilde{\pi}$ falls, causing the effective work force that must be supplied with capital to rise and lowering physical capital intensity, other things constant.

In summary, the overall effect of the structural transformation on physical capital intensity is ambiguous. The positive effects stemming from changes in saving and fertility are potentially offset by the negative effects of the increasing physical capital requirement and an increase in adult equivalent labor supply per worker.

D. Labor Productivity

As discussed in the introduction, we need to examine the behavior of two different measures of worker productivity. Labor productivity, defined as aggregate output *per hour worked*, is given by

(16a) $$y_t^{hour} = A_t^{\frac{1}{1-\alpha}} k_t^{\alpha} h_t \bar{m}_t^{hour},$$

where $\bar{m}_t^{hour} \equiv \dfrac{\tilde{\pi}_t \tilde{m} f + (1-\tilde{\pi}_t) m}{\tilde{\pi}_t (1-\tilde{n}_t(\tau-(T-e_t)) f + (1-\tilde{\pi}_t)(1-n_t(\tau-(T-e_t))}$, is the adult equivalent labor supply per hours worked. Labor productivity defined as aggregate output per *worker*, is given by

(16b) $$y_t^{worker} = A_t^{\frac{1}{1-\alpha}} k_t^{\alpha} h_t \bar{m}_t^{worker},$$

where $\bar{m}_t^{worker} \equiv \dfrac{\tilde{\pi}_t \tilde{m} f + (1-\tilde{\pi}_t) m}{\tilde{\pi}_t (1+\tilde{n}_t) + (1-\tilde{\pi}_t)(1+n_t)}$, is the adult equivalent labor supply per worker. An important question is whether the structural transformation from traditional to modern production can have a significant impact on (16b), while having little impact on (16a).

IV. Quantitative Analysis

In this section we calibrate the parameters of the models and then test the resulting quantitative model against the remaining historical observations not used in the calibration. The section ends with a theoretical growth accounting.

A. Calibration

The time periods are interpreted to last 20 years. The productive time endowment of children for work or school, *T*, is set to 0.5 (one half the time endowment of a young adult). One can think of the child has having no capability for productive activity from ages 0 to 4, half that of an adult from ages 5 to 14, and the same capabilities for productive effort as an adult from ages 15 to 19. We assume τ = 0.16, a value similar to those calibrated in other studies (e.g. Doepke (2004) and Lord and Rangazas (2006), and Mourmouras and Rangazas (2007)).

The steady state is assumed to be characterized by full-time schooling, $s = \tilde{s} = T$, and a fertility rate of one child per parent in landless households, $n = 1$. The initial fertility of landless households in 1800 is assumed to be 2.4, the fertility rate for urban households in 1800, per adult (see Greenwood and Seshadri (2002)). Lord and Rangazas (2006) argue that time spent in school, as a fraction of total days in the year, was in the range 0.08 to 0.10 during the second half of the 19^{th} century, and increased only modestly before then. We assume that the lower-bound schooling level for the economy and the amount of schooling received by young children are both 0.08. These two steady state assumptions and two initial conditions are used to simultaneously calibrate four parameters: γ = 0.2250, θ = 0.5110, ε = 0.5403, and ψ = 0.2693. The value for γ has also been estimated directly by historians. Our estimate is remarkably close to the value of 0.229 estimated by Lebergott (1964, pp.49-50) by comparing farm wages of 10 year olds relative to adults in 1798.

The initial value of schooling, received by the generation of households who become young adults in 1800, was set so that the dynamic process for schooling (see

(7b)) generates a match to the observed schooling level in 1880. This year was one of the earliest census-based estimates of schooling and was also a sufficient distance from the Civil War period, so that the schooling level was not unduly affected by the war. Approximately the same initial schooling level would have been calibrated had we targeted any value for schooling in the second half of the 19th century (i.e. any year after schooling was first estimated by the census starting in 1850).

The production technologies are calibrated based on the parameter values chosen by Hansen and Prescott (2002): α = 0.40, ρ = 0.30, and $\phi = 0.10$. To simplify some of the derivations and computations we set δ = 1, i.e. complete depreciation of physical capital over the 20 year period. We set the steady state return to capital so that the simulated annual rate of return to capital was 7 percent in 2000—the approximate value of the return to physical capital in the United States, whether proxied by the trendless rate of return to the Standard and Poor's 500 during the 20th century (Kocherlakota (1996)) or estimated by combining direct methods to compute the marginal product of capital (Caselli and Feyrer (2005)) with estimates of an annual rate of physical depreciation of 6 to 7 percent (e.g. Rebelo and Stokey (1995))). The resulting steady state annualized rate of return is 5.7 percent. The steady-state physical capital intensity, consistent with this rate of return, generates a calibrated value for β equal to 0.4136.

We set the initial physical capital intensity based on observed differences in the return to assets across the two centuries. Stock market returns and direct measures of the return to capital do not go back as far as 1800. However, there is evidence that interest rates were 2 to 5 percentage points higher in the 19th century than in the 20th century. Wallis (2000, Figure 2) reports that real interest rates on national government debt averaged about 5% in the first half of the 19th century and averaged about 2.5 % in the 20th century. Barro (1993) reports that real interest rates on commercial paper were 9% from 1840 to 1880, but were about 4 % at the beginning and end of the 20th century. The setting of the initial *k* was also guided by the attempt to keep the rate of return to capital in 1800 close to the endogenous value in 1820 (so that the chosen initial capital intensity is consistent with the fundamentals of the model). Guided by this consideration, and the historical interest rate decline, we set the initial *k* so that the return to physical in 1800 was 10 percent, or 3 percentage points above the rate of return in 2000.

An important focus of our study is the fraction of the period worked by landed households, *f*. *Proposition 2* and (9a) can be used to link *f* to the fertility of landed households. In equilibrium, land rents must compensate landed households for a shorter paid work-year. Land rents, in turn, determine the fertility of landed households. We estimate the fertility of landed households in 1800 by using the observed historical fertility of rural households—3.6 children per adult (Greenwood and Seshadri (2002)). To match the fifty percent higher fertility of landed (rural) households compared to landless (urban) households, Ω must be 0.50. Given the calibrated values of ψ and β, (10) then implies that $f = 0.62$. This value should be interpreted as the shorter work year of "full-time" landed adult worker relative to a "full-time" landless adult worker. David (2005) estimates *f* to be a somewhat higher value, 0.76.

We can also compare the unconstrained adult-equivalent labor supply of *all* workers in landed and landless households. This comparison captures the fact that the labor supply of households falls short of full-time because of childrearing and because the labor supply of children falls short of the supply of adults (due to the fewer hours they supply, as a result of schooling, and because their productivity is less than an adult). The calibrated parameters and *Proposition 1* imply that the adult equivalent labor supply of landed households is about 90 percent of landless households. Combining both the unconstrained labor supply differences and the seasonal constraints on full-time work, implies that a landed household works only 56 percent as much as a landless household does. David (2005) estimates that agricultural workers in 1840 worked 52 percent as much as nonagricultural workers over the course of a year. While the labor supply concept used by David is not exactly the same as ours, it is encouraging that the differences in work across households are in the same ballpark.

We need to set the growth of exogenous technological change in the modern sector and the relative growth of exogenous technological change in the traditional sector. The growth in exogenous technological change in the modern sector was set to match the growth in worker productivity (on an hourly basis) over the two centuries. Worker productivity per hour grew at an annual rate of 1.37% from 1800 to 2000 and we found that an exogenous growth rate in TFP of 0.33% was needed to match it.

Finally, the time series for $\tilde{a}_t \equiv \frac{\tilde{A}_t}{A_t}$ is set to match the fraction of young households that are landed in each period.[6] We estimate this fraction by assuming that (i) the fraction of workers employed in the traditional sector is equal to the observed fraction of labor employed in agriculture (*agr.share*) and (ii) the ratio of households that operate the traditional technology to the labor they employ is equal to the historical ratio of farms to agricultural labor (*agr.labor*).[7] Under these assumption we have

$$\frac{\tilde{N}_{t-1}}{agr.share_t(\tilde{N}_t + N_t)} = \left(\frac{farms}{agr.labor}\right)_t,$$

which implies

$$\tilde{\pi}_{t-1} = agr.share_t \times \left(\frac{farms}{agr.labor} \right)_t \times avg.\,fertility_{t-1}, \tag{17}$$

where *avg.fertility* is the *observed* average fertility per adult (Haines (2004, Table 4)). Table 3 gives the estimated values for $\tilde{\pi}_t$.

Equation (11) can be written as

$$\frac{\tilde{a}_{t+1}}{\tilde{a}_t} = \left[\frac{\tilde{\pi}_t}{\tilde{\pi}_{t-1}} \frac{h_t}{h_{t-1}} \left(\frac{k_t}{k_{t-1}} \right)^{\alpha} \bar{n}_{t-1} \left(1+g\right)^{\frac{1}{1-\alpha}} \right]^{\rho} . \tag{18}$$

Given Table 3 and the simulated values for the economy's transition, we can then back-out the implied growth in the relative TFP of the traditional sector.

This completes the calibration. The calibrated parameters and initial conditions are summarized in Table 4. The implied series given by (18) will be discussed when we compare the simulation to the actual data in the next section.

B. Testing

The calibrated models can now be used to simulate the economic development of the U.S. from 1800 to 2000. The predictions of the models are compared to actual data not used in the calibration. The data free for testing includes (i) the path of schooling during the 20th century, (ii) the pattern of the fertility decline over both centuries, (iii) the pattern of the decline in the rate of return to capital over the two centuries, (iv) the difference in the average rate of growth of worker productivity across the 19th and 20th centuries, (v) the difference in the growth rate of worker productivity on a per hour basis versus a per worker basis, and (vi) the relative growth in traditional technology/land improvement over both centuries.

Schooling

Figure 1 compares the predicted schooling to actual schooling. The model matches schooling almost exactly until 1920. In the 19th century, neither the predicted nor the actual schooling rises much despite the substantial reallocation of households from the traditional to the modern sector. This supports the theory given by (9b) where schooling is independent of land ownership. The lack of connection between the structural transformation and schooling also helps explain why there was slow growth in labor productivity per hour worked in the 19th century.

At the beginning of the 20th century, schooling begins to rise sharply, in the model and in the data. In the middle of the 20th century, the predicted schooling does not rise as fast as actual schooling, although it matches actual schooling well by century's end. The failure to match the rapid rise in schooling in the middle of the century is, in part, due to government's expanding subsidy of schooling during the high school movement and then after WWII through programs such as the GI Bill and Pell Grants, policies that are missing from the model.

Fertility

As seen in Figure 2, the model matches the fertility decline over the two centuries fairly closely. Since the rise in schooling was very weak in the 19th century, the majority of the simulated decline is due to the movement of households away from traditional production on family farms. The rise in schooling became an important source of the fertility decline during the 20th century. There are certainly some missing factors that would help explain the accelerated fertility decline at the end of the 19th and beginning of the 20th century that are not captured by the model—for example, declining child mortality.

Rates of Return to Physical Capital

As displayed in Figure 3, the model predicts a gradual decline in interest rates starting around the turn of the century. The starting point of the decline is reasonably accurate, but instead of a gradual decline, actual interest rates fell sharply between 1870 and 1910 and then showed no further trend. The sharp drop in interest rates was at least in part due to an inflow of foreign capital during the peak of the U.S. industrial revolution. Since we assume a perfectly closed economy, this capital inflow is not captured here.

The fact that the model (and the data) display relatively constant rates of return over most of the 19th century, suggests that the structural transformation kept physical capital intensity from rising toward its higher steady state value. To explain the relatively constant rates of return, recall the discussion of *Proposition 4*. The greater numbers of workers that had to be equipped with physical capital, as workers shifted out of the traditional sector and into the more capital intensive modern sector, more than offset the increasing aggregate saving rate and declining population growth rate. We say more than offset because, given that we set the initial value of k below its steady state value, the tendency should be for k to rise and r to fall, other things constant. As with schooling, the relatively constant physical capital intensity helps explain the slow growth in labor productivity per hour worked.

Worker Productivity (per hour)

Figure 4 compares the actual growth rates in output per hour worked to those predicted by the model (i.e. after adjusting for the seasonal constraint on hours worked by landed households and the hours lost to childrearing and schooling). The rate of exogenous technological change was calibrated to match the average growth rate in worker productivity over the entire two centuries. Thus, the interesting issue is the extent to which the model can explain the *rise* in growth rates over the two centuries. As can be seen in the figure, the model does predict rising growth rates, but it fails to explain the full rise from the very low rates at the beginning of the 19th century to the very high rates in the middle of the 20th century. In the actual data, the average annual growth rate was 0.78% in the 19th century and 1.95% in the 20th century, a difference of 1.17 percentage points. The model predicts growth rate of 1.03% in the 19th century and 1.70% in the 20th century, a difference of 0.67 percentage points or 57% of the actual difference. Thus, the majority of the rise in growth rates is explained by the model, but the assumption that exogenous technological progress was constant throughout the two centuries causes the growth rate to be ¼ of a percentage point too high in the 19th century and ¼ of a percentage point too low in the 20th century.

Worker Productivity (per worker)

Figure 5 compares the actual growth rates in output per worker to those predicted by the model, where output is now simply divided by the number of adults and children in the work force—with no adjustment for how much they work. The actual growth rates are tracked reasonably well on average over the two centuries (this is not guaranteed since the exogenous rate of technological change in the modern sector was set to match the average growth in productivity *per hour worked* and not *per worker*). The actual growth rate in productivity per worker during the 19th century was 1.36% and the model predicts

1.45%. In the 20th century the actual growth rate was 1.75% and the model predicts 1.83%.

A focus of our study is the large difference in the growth rate per worker and growth rate per hour worked in the 19th century—0.58 percentage points. The model can explain a difference of 0.42 percentage points or 72% of the actual difference. In the model, the difference is generated by the fact that the structural transformation does *not* affect productivity growth *per hour worked* (as discussed above, schooling and physical capital intensity were unaffected by the structural trasnformation), but it clearly *raises* productivity *per worker* by increasing the hours worked per worker.

Farm Technology/Land Improvement

In Figure 6, we plot the growth in relative TFP in the traditional sector as given by (18). Throughout the 19th century the growth in $\tilde{a}_t$ had to be high to absorb the increasing number of workers in the traditional sector. The predicted annual average growth rate in $\tilde{a}_t$ was 1.48%, which we interpret as primarily being the result of land expansion and improvement. After 1900, the average estimated growth in $\tilde{a}_t$ was close to zero.

The predicted pattern of $\tilde{a}_t$-growth qualitatively matches the history of land expansion and improvements in the U.S.. Land did expand rapidly in the 19th century U.S., during the “cheap land” era, where land could be acquired at no or little price (Benedict (1953, pp.12-22) and Saloutos (1962)). Gallman (1992) estimated that the stock of improved-land increased about 10-fold over this century. While land used in farming continued to expand until 1950 (Gardner (2002, Figure 1.1)), it slowed considerably after 1900 when it became difficult to obtain good land cheaply or improve it without great effort (Garner (2002, p.3), Benedict (1953, p.112) and Saloutos (1962, p.449-450)).

Given the simulated series for $\tilde{a}_t$, we used (12) to confirm that the labor marker did remain integrated through the two centuries of the simulation.

C. Theoretical Growth Accounting

Our empirical application ends with a *theoretical* growth accounting of the sources of growth over the two centuries based on (16). The accounting is considered "theoretical" for two reasons. First, in some cases, the growth rates that are decomposed are the rates *predicted* by the model and not the *actual* growth rates—for example, the actual and predicted rates will differ in the accounting *within* centuries. Second, the contribution of each determinant is causal, unlike in the traditional growth accounting. In particular, our measures of factor accumulation are independent of exogenous technological change.

Table 5 decomposes the sources of the growth in worker productivity, per hour and per worker, over each century and over the entire period. The easiest case to interpret is the accounting for the growth in worker productivity per *hour* over the two centuries (the second to last row in table). This is the case where the rate of exogenous technological change is chosen to match the actual historical growth rate. For this case TFP growth explains 41 percent of the growth, with factor accumulation explaining 59 percent. Human capital accumulation and increases in the adult equivalent labor supply per hour worked explain ¾ of the growth that is due to factor accumulation.

The predicted growth rate in productivity per *worker* over the two centuries, 1.64 percent per year, is very close to the actual 1.56 percent per year. Thus, the last row gives a very close accounting of the actual growth rate. In this case, increases in TFP explain 34 percent of growth. This is close to the value of 39 percent TFP growth estimated by Baier et al (2006) using a standard (atheoretical) growth accounting for the period 1870 to 2000. For productivity per worker, human capital and adult-equivalent labor supply explain 82 percent of the growth due to factor accumulation and 54 percent of the growth overall.

The growth accounting for each century is harder to interpret because the predicted growth rate for the 19th century is ¼ percentage point too high and the predicted growth rate for the 20th century is a ¼ percentage point too low. However, in the 19th century, the predicted increase in human and physical capital accumulation are quite low and the predicted increase in adult-equivalent labor supply is relatively high, especially on a per worker basis. This is an important result because it helps to explain why growth in output per hour was relatively low, and growth per worker was relatively high, in the 19th century. In addition, the predicted growth in both human capital and adult-equivalent labor supply are *independent* of the chosen rate of exogenous technological progress because of the recursive structure of the theoretical model. The portions of the *actual* 19th century growth rates explained by human capital and adult-equivalent labor supply are 57 percent for productivity per hour and 63 percent for productivity per worker.

V. Conclusion

This paper uses the difference between rural and urban fertility at the beginning of the 19th century to estimate the ratio of land rents to wages for traditional farmers in the United States. The estimated land rent-wage ratio implies a short-fall in annual hours worked by traditional farmers relative to landless households who sell their labor for hire. We estimate that traditional farmers supply 56 percent less effective labor than landless households, thus explaining an almost 2-fold annual wage gap between traditional agriculture and other sectors of the economy.

The model was used to quantify the effect on aggregate growth of labor moving out of traditional farming. Consistent with the data, we find that the structural transformation in the United States had little effect on labor productivity per *hour worked* because the structural transformation had little effect on human and physical capital intensity per worker. However, the structural transformation had a substantial effect on labor productivity per *worker* because it caused a rise in adult-equivalent hours worked per year.

While our study focused on United States history, we think it is likely that there are relatively low hours worked in traditional agriculture generally. Thus, differences in hours worked may be an important reason for the large wage/ productivity gaps observed across agriculture and industry in today’s developing countries.

APPENDIX—Proofs of the propositions.

Proposition 1

Write m_t as $1-n_t\left(\tau-\gamma\frac{\bar{h}}{h_t}(T-e_t)\right)$. Using (7a), note that

$n_t\left(\tau-\gamma\frac{\bar{h}}{h_t}(T-e_t)\right)=\frac{\psi}{1+\beta+\psi}$. Substituting into the expression for m_t above gives

$m_t=\frac{1+\beta}{1+\beta+\psi}$. Using (9a) in similar fashion gives the expression for $\tilde{m}_t$.

Proposition 2

We begin by forming the indirect lifetime utility function for both landed and landless households. In equilibrium, the lifetime utility of the two household types must be equal. Equating the indirect utility functions of the two types, and eliminating common terms gives, $\ln\left[\frac{1}{f}\right]=\ln\left[\left(1+\frac{\rho O_{t+1}}{R_t W_t f\tilde{h}_t}\right)^{\frac{1+\beta+\psi}{1+\beta}}\right]$. Solving for $\frac{\rho O_{t+1}}{R_t W_t f\tilde{h}_t}$, then gives (10).

Proposition 3

First, note that labor's share of traditional output can be written as $\frac{W_{t+1}\tilde{H}_{t+1}}{O_{t+1}}=1-\rho-\phi$.

Using (10) then implies, $\Omega=\frac{\rho}{1-\phi-\rho}\frac{W_{t+1}\tilde{H}_{t+1}}{R_t W_t f\tilde{h}_t}$. Solving for $\tilde{H}_{t+1}$, using (2) and the assumption that $\tilde{h}_t=h_t$, gives

(A1) $$\tilde{H}_{t+1}=\frac{(1-\rho-\phi)\alpha f\Omega h_t k_t^{\alpha}}{\rho k_{t+1}(1+g)^{\frac{1}{1-\alpha}}}.$$

Next, using the equality in (11) and (A1), we can solve for $\tilde{N}_t$ as

(A2) $$\tilde{N}_t = \frac{\rho\left(f\tilde{a}_{t+1}\right)^{\frac{1}{\rho}}\left(\frac{\phi}{\alpha}\right)^{\frac{\phi}{\rho}}}{(1-\phi-\rho)\alpha f\Omega h_t k_t^{\alpha} A_t^{\frac{1}{1-\alpha}}}.$$

The inequality we seek to satisfy is $\tilde{N}_t\tilde{H}_{t+1} \geq \tilde{N}_{t+1} fh_{t+1}\tilde{m}$ or

(A3) $$\tilde{H}_{t+1} \geq \frac{\tilde{N}_{t+1}}{\tilde{N}_t} fh_{t+1}\tilde{m}.$$

Substituting (A1) and (A2) into (A3), rearranging and then using (2b), yields (12).

Proposition 4

We begin by working with the left-hand-side (LHS) of (13), rewriting it as,

$K_{t+1} + \tilde{K}_{t+1} = A_{t+1}^{\frac{1}{1-\alpha}}\left(k_{t+1}H_{t+1} + \tilde{k}_{t+1}\tilde{H}_{t+1}\tilde{N}_t\right)$. Using (2), (4), and the fact that $\alpha = \phi + \rho$, gives us

(A4) $$\tilde{k}_t = \frac{\phi}{\alpha}k_t.$$

Also, H_{t+1}, the supply of human capital to the modern sector, is the total supply of human capital to the economy minus the human capital demanded by the traditional sector,

(A5) $$H_{t+1} = N_{t+1}mh_{t+1} + \tilde{N}_{t+1}\tilde{m}fh_{t+1} - \tilde{N}_t\tilde{H}_{t+1}.$$

Substituting (A3) and (A4) in the RHS of (A5) gives us

$$A_{t+1}^{\frac{1}{1-\alpha}}(N_{t+1}m + \tilde{N}_{t+1}\tilde{m}f)h_{t+1}k_{t+1} - \left(\frac{\alpha-\phi}{\alpha}\right)\left(\frac{\phi}{\alpha}\right)^{\frac{\phi}{\rho}}\left(f\tilde{a}_{t+1}\right)^{\frac{1}{\rho}}.$$

Using (A2), the last part of the expression above can be rewritten to yields the following expression for the aggregate demand for capital

(A6) $$K_{t+1}+\tilde{K}_{t+1}=A_{t+1}^{\frac{1}{1-\alpha}}\left(N_{t+1}m+\tilde{N}_{t+1}\tilde{m}f\right)h_{t+1}k_{t+1}-\tilde{N}_t\,\frac{\alpha-\phi}{\rho}\,f\Omega A_t^{\frac{1}{1-\alpha}}h_t k_t^{\alpha}\,.$$

Now, we focus of the RHS of (13). The savings functions, (7c) and (9c), along with (10), allow us to write the RHS as $\tilde{N}_t\tilde{s}_t+N_t s_t=\left(\tilde{N}_t\Gamma f+N_t\beta\right)\frac{W_t h_t}{1+\beta+\psi}$, where $\Gamma\equiv\beta-\left(1+\psi\right)\Omega$. Substituting from (2a) then allows us to write the RHS as

(A7) $$\tilde{N}_t\tilde{s}_t+N_t s_t=\left(\tilde{N}_t\Gamma f+N_t\beta\right)\frac{A_t^{\frac{1}{1-\alpha}}(1-\alpha)k_t^{\alpha}h_t}{1+\beta+\psi}\,.$$

Equating (A6) and (A7), rearranging, and defining $\tilde{\pi}_t\equiv\tilde{N}_t/(N_t+\tilde{N}_t)$, yields (14).

FOOTNOTES

1. The link between the farm and the family was surprisingly strong well into the 20th century. At the turn of the century, the farm in the US was still largely operated individually and organized around the family. Most of farm labor was provided within the family. Even by 1930 only 42 percent of all farms reported hiring labor outside the family (Ely and Werwein (1940, p.162)). As late as 1978, traditional family farms represented 88 percent of all farms accounting for 63 percent of total farm production (Gardner (2002, pp.56-57)). Intergenerational links to farming seem to be a common feature of many developing countries for a variety of reasons. Hayashi and Prescott (2006) claim that Japan's development was slowed by a social convention of passing the farm along within the family. The paper includes a passage (p.40), suggested by Andrew Foster, arguing that the social convention may be strong enough to operate even in the presence of a land market. "First, the heir could sell the inherited farmland and live in the city to collect the higher urban income. However, to prevent this, the father could require the son to remain on the farm until he inherits the land. By the time his son inherits the estate, it may be too late for him to start a career in the city." Collier, Radwan, and Wange (1986) find that those individuals who indefinitely migrate away from farms in Tanzania tend to lose their land entitlement. In China, explicit migration rules cause migrants to the city to give up ownership claims to land and small businesses in rural areas (Au and Henderson (2006)).

2. During the 19th century, Primack (1969) estimates that farmers spent about 20 percent of their annual work hours in activities such as improving land, constructing farm buildings, and fencing property. We assume that land improvements and maintenance are exogenous and captured in farm TFP. For an interesting and detailed first attempt to make land improvements endogenous see Vandenbrouke (2003, 2004). We also assume that such activities did *not* increase the net flow of income to the current generation, but did increase the quantity of effective land bequeathed to the next generation.

3. Unlike some approaches, we follow Schultz (1964) and assume that human capital is productive on the farm. There is a good deal of evidence supporting this stance (e.g. Jamison and Lau (1982), Lucas (1985), Foster and Rosenzweig (1996), Goldin and Katz (2000), Duflo (2001), and Jenkins and Knight (2005)).

4. Mourmouras and Rangazas (2007) examine a "poverty trap" outcome with $e_t = \overline{e}$ for all *t*.

5. Schooling differences across sectors have never been very large. Based on the 1915 Iowa census, Goldin and Katz (1999) find the average years of schooling in white collar occupations was 10.8 years, while in blue collar occupations and farming it was 7.8. Using the 1960 US census, Murphy and Welch found the average years of schooling were 13.4 in white collar occupation, 9.8 in blue collar occupations, and 9.1 in farming.

6. Direct quantitative estimates of TFP growth in agriculture and nonagricultural sectors over this periods can be found in Atack, Bateman, and Parker (2000) and Greenwood and Seshadri (2002). Unfortunately these estimates are flawed because Atack et al interpreted Weiss's (1993) *labor productivity* estimates as *TFP* estimates (see also Mundlak (2005, footnote 30)). Greenwood and Seshadri based their calculations on Atack et al's interpretation, so their estimates suffer from the same problem. Beyond this error, any attempt to calculate TFP growth over this period faces the difficulty of measuring physical and human capital accumulation during the 19th century.

7. The number of farms is taken from the *Historical Statistics of the United States* (Colonial Times to 1970), series K1-16. The number of agricultural workers is based on David (2005) and Kendrick (1961, 1973).

References

Abramovitz, M., and David, P., 2001, “Two Centuries of American Macroeconomic Growth, SIEPR Discussion Paper, No. 01-05.

Atack, J., Bateman, F., and Parker, W., 2000, “The Farm, the Farmer, and the Market.” In Engerman, S. and Gallman, R. (editors), *The Cambridge Economic History of the United States Volume 2*. Cambridge: Cambridge University Press.

Au, C., and Henderson, J., 2006, “How Migration Restrictions Limit Agglomeration Production in China,” *Journal of Development Economics*, 80, 350-388.

Baier, S., Dwyer, G., and Tamura, R., 2006, “How Important are Capital and Total Factor Productivity for Economic Growth,” *Economic Inquiry*, 44, 23-49.

Benedict, M., 1953, *Farm Policies of the United States 1790-1950*. New York: The Twentieth Century Fund.

Broadberry, S. and Irwin, D., 2006, “Labor Productivity in the United States and the United Kingdom during the Nineteenth Century,” *Explorations in Economic History*, 43, 257-279.

Carter, S., Ransom, R., and Sutch, R., 2003, "Family Matters: The Life-cycle Transition and the Unparalleled Fertility Decline in Antebellum America." In T. Guinnane, W. Sundstrom, and W. Whately (Eds.), *History Matters: Essays on Economic Growth, Technology, and Demographic Change*. Stanford: Stanford University Press.

Caselli, F., 2005, "Accounting for Cross-Country Income Differences, in D. Aghion and S. Durlauf, eds., *Handbook of Economic Growth*, 679-741, Amsterdam: Elsevier.

Caselli, F. and Coleman, W., 2001, "The US Structural Transformation and Regional Convergence: A Reinterpretation," *Journal of Political Economy*, 109, 584-616.

Caselli, F., and Feyrer, 2005, "The Marginal Product of Capital," NBER Working Paper #11551.

Collier, P., Radwan, S., and Wangwe, S., 1986, *Labour and Poverty in Rural Tanzania: Ujameen and Rural Development in the United Republic of Tanzania*. New York: Oxford University Press.

Cordoba, J., and Ripoll, M., 2006, "Agriculture, Aggregation, Wage Gaps, and Cross-Country Income Differences," Mimeo

David, P., 2005, "Real Income and Economic Welfare Growth in the Early Republic," Stanford University Department of Economics Working Paper 06-001.

Doepke, M., (2004), “Accounting for the Fertility Decline during the Transition to Growth,” *Journal of Economic Growth*, 9(3), 347-383.

Drazen, A. and Eckstein, Z., 1988, “On the Organization of Rural Markets and the Process of Development,” *American Economic Review*, 78, 431-443.

Duflo, E., 2001, “Schooling and Labor Market Consequences of School Construction in Indonesia: Evidence from an Unusual Policy Experiment,” *American Economic Review*, 91, 795-813.

Ely, R., and Wehrwein, G., 1940, *Land Economics*. New York: Macmillan Co.

Foster, A., and Rosenzweig, M., 1996, “Technical Change and Human Capital Returns and Investments: Evidence from the Green Revolution,” *American Economic Review*, 86, 931-953.

Gallman, R., 1992, “American Economic Growth before the Civil War: The Testimony of Capital Stock Estimates.” In R. Gallman and J. Wallis, editors, *American Economic Growth and Standards of Living before the Civil War*, Chapter 2. Chicago: University of Chicago Press.

Galor, O., 2005, “From Stagnation to Growth: Unified Growth Theory.” In P.Aghion and S. Durlauf, (Eds.), *Handbook of Economic Growth*. Amsterdam: North Holland.

Galor, O. and Weil, D., 2000, "Population, Technology, and Growth: From Malthusian Stagnation to Demographic Transition and Beyond," *American Economic Review*, 90, 806-828.

Galor, O. and Weil, 1996, "The Gender Gap, Fertility, and Growth," *American Economic Review*, 86, 374-387.

Gardner, B., 2002, *American Agriculture in the Twentieth Century*. Cambridge Mass: Harvard University Press.

Goldin, C. and Katz, L., 1999, "The Returns to Skill in the United States Across the Twentieth Century," NBER Working Paper #7126.

__________________, 2000, "Education and Income in the Early 20th Century: Evidence from the Praires," *Journal of Economic History*, 782-818.

Gollin, D., Parente, S., and Rogerson, R., 2004, "Farmwork, Homework, and International Income Differences," *Review of Economic Dynamics*, 74, 827-850.

Greenwood, J. and Seshadri, A., 2002, "The US Demographic Transition," *American Economic Review* (Papers and Proceedings), 92, 153-159.

Habbakuk, J., 1962, *American and British Technology in the Nineteenth Century*. Cambridge: Cambridge University Press.

Haines, M., 2000, "The Population of the United States 1790-1920." In S. Engerman and R. Gallman (Eds), *The Cambridge Economic History of the United States*. New York: Cambridge University Press.

Harris, J., and Todaro, M., 1970, "Migration, Unemployment, and Development: A Two-Sector Analysis," *American Economic Review*, 66, 126-142.

Hatton, T., and Williamson, J., 1992, "What Explains Wages between the Farm and City? Exploring the Todaro Model with American Evidence, 1890-1941," <u>Economic Development and Cultural Change</u>, 40, 267-294

____________, 1991, "Integrated and Segmented Labor Markets: Thinking in Two Sectors," *Journal of Economic History*, 51, 413-425.

Hayashi, F.,and Prescott, E., 2006, "The Depressing Effect of Agricultural Institutions on the Pre-War Japanese Economy," *NBER Working Paper* #12081.

Hazan, M., and Berdugo, B., 2002, "Child Labor, Fertility, and Economic Growth," *Economic Journal*, 112, 810-878.

Jamison, and Lau, L., 1982, *Farmer Education and Farm Efficiency*, Baltimore: Johns Hopkins University Press.

Jenkins, C., and Knight, J., 2002, *The Economic Decline of Zimbabwe*. New York: Palgrave Press

Kendrick, J., 1961, *Productivity Trends in the United States*. Princeton: Princeton University Press.

_____________, 1973, *Postwar Productivity Trends in the United States 1948-1969*. New York: Columbia University Press.

Kuznets, S., 1965, *Modern Economic Growth: Rate Structure, and Spread*. New Haven: Yale University Press.

Lord, W., and Rangazas, P., 2006, Fertility and Development: The Roles of Schooling and Family Production, *Journal of Economic Growth*, 11, 229-261.

Lucas, R.E.B., 1985, “Migration Amongst the Batswana,” *Economic Journal*, 95, 358-382.

Margo, R., 2000, *Wages and Labor Markets in the United States 1820-1860*. Chicago: University of Chicago Press.

Moav, O., 2005, Cheap Children and the Persistence of Poverty, *Economic Journal*, 115, 88-110.

Mourmouras, A., and Rangazas, P., 2007, Foreign Aid Policy and Sources of Poverty: A Quantitative Framework, *IMF Staff Papers*, 54, (forthcoming).

Mundlak, Y., 2005, “Economic Growth: Lessons from Two Centuries of American Agriculture,” *Journal of Economic Literature*, 43, 989-1024.

Murphy, K., and Welch, F., 1993, Occupational Choice and the Demand for Skill, 1940-1990, *American Economic Review*, 83, 122-126.

Primack, M., 1969, Farm Fencing in the Nineteenth Century, *Journal of Economic History*, 29, 287-291.

Rebelo, S., and Stokey, N., 1995, “Growth Effects of Flat-Rate Taxes,” *Journal of Political Economy*, 103, 519-550.

Restuccia, D., Yang, D., and Zhu, X., 2004, “Agriculture and Aggregate Productivity: A Quantitative Cross-Country Analysis,” Mimeo

Saloutos, T., 1962, “Land Policy and Its Relation to Agricultural Production and Distribution, 1862 to 1933, *Journal of Economic History*, 22, 445-460.

Schultz, T., 1964, *Transforming Traditional Agriculture*. New Haven: Yale University Press.

Sicsic, P., 1992, “City-Farm Wage Gaps in the Late Nineteenth Century France,” *Journal of Economic History*, 52, 675-695.

Vandenbrouke, G., 2003, “The U.S. Westward Expansion,” *Economics d’avant garde, Research Report No. 4.*

______________, 2004, “The American Frontier: A Hundred Years of Western Settlement,” *Economics d’avant garde, Research Report No. 7.*

Wallis, J. , 2000, “American Government Finance in the Long-Run: 1790 to 1990,” *Journal of Economic Perspectives*.

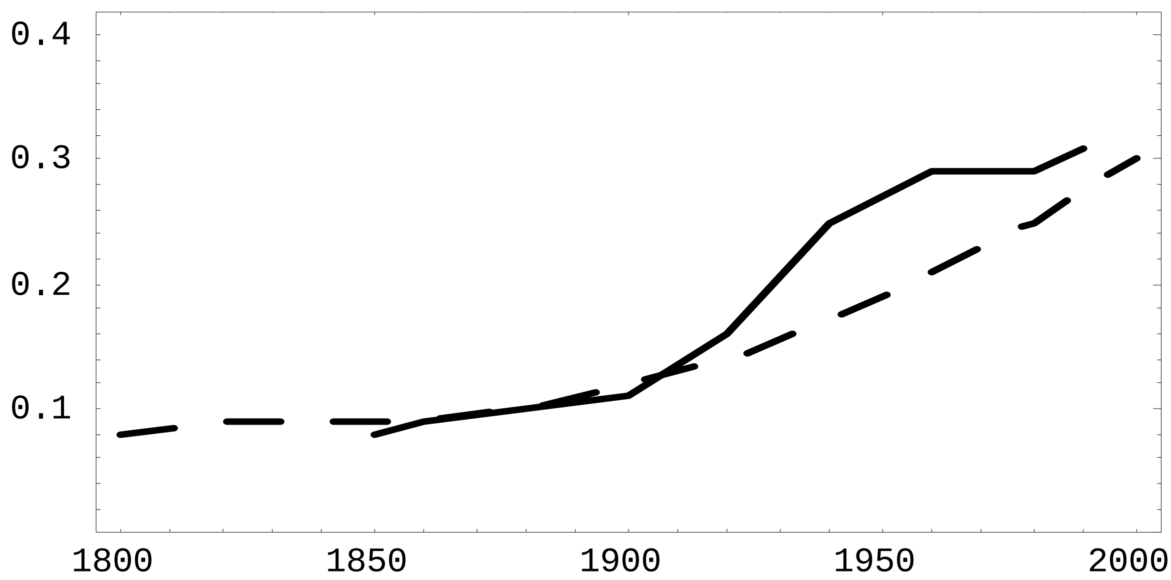

Figure 1 U.S. Schooling 1800 to 2000
Source: Solid-plot is actual schooling from Lord and Rangazas (2006, Table 2). Dashed-plot is simulated schooling

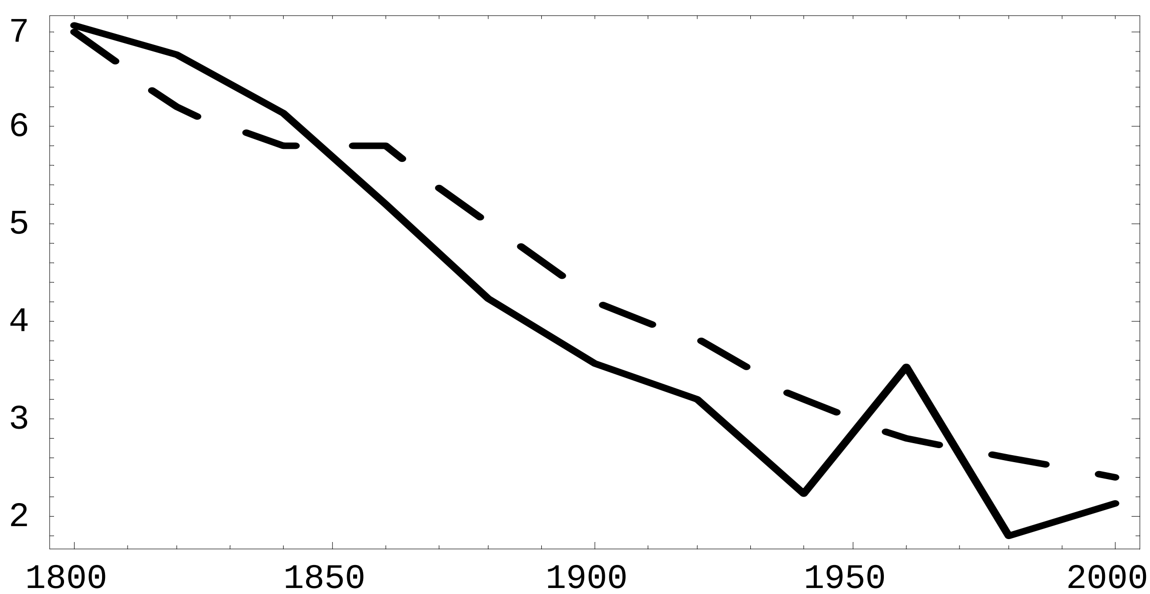

Figure 2 U.S. Fertility 1800 to 2000
Source: Solid-line is the actual TFR from Haines (2000, Table 4). Dashed-line is the simulated fertility of the model multiplied by 2 to place it on a per woman basis.

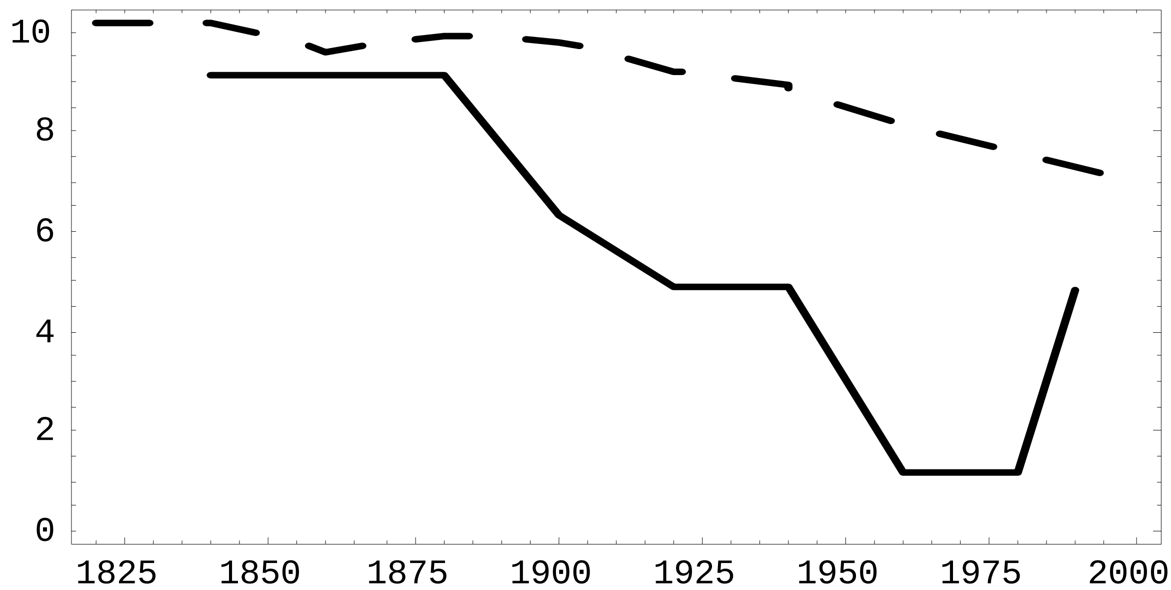

Figure 3 Rate of Return to Physical Capital 1800-2000
Sources: Interest rates in annual percent. Solid line are actual interest rates on six-month prime commercial paper from Barro (1997, Table 11.1). Simulated rates of return are given by the dashed-line.

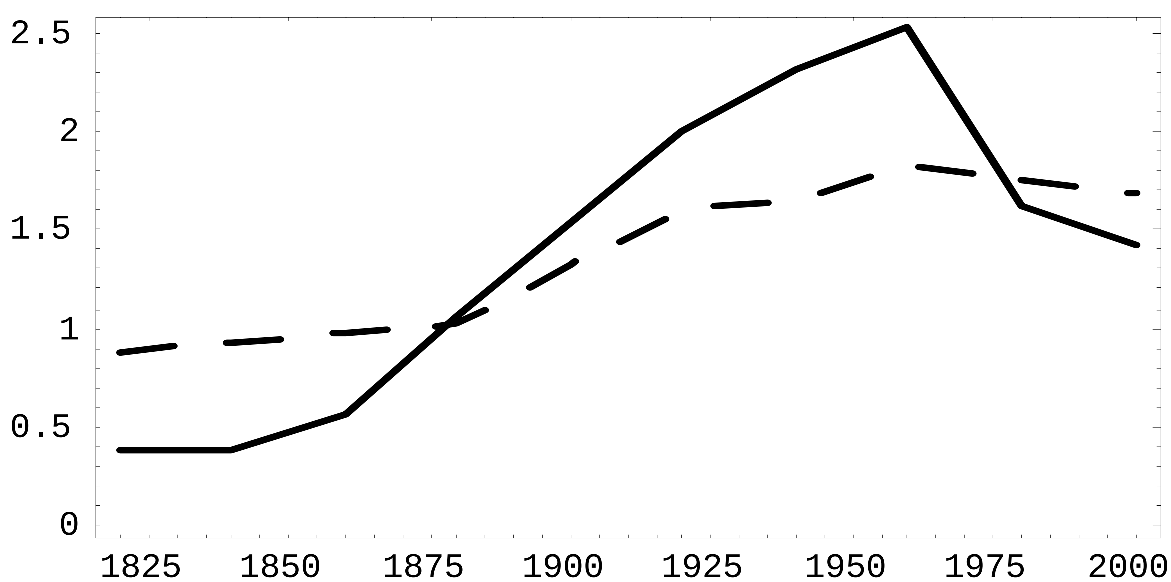

Figure 4 Worker Productivity Growth Rates (Per Hour Worked)
Sources: The solid-line is the actual growth in productivity per hour worked (see Table 1). The dash-lined gives the simulated growth rates.

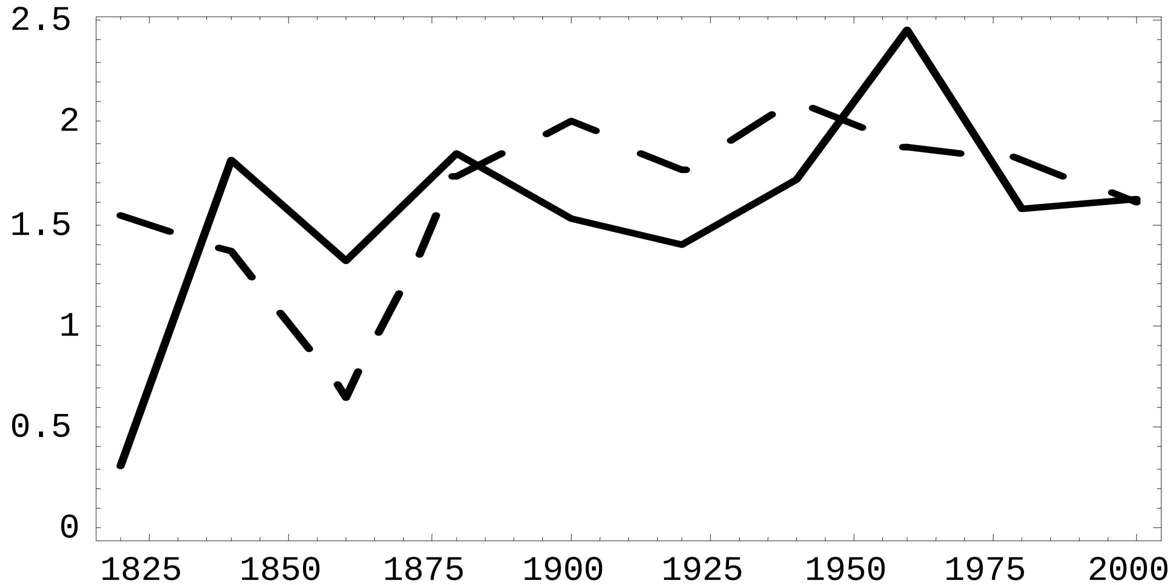

Figure 5 Worker Productivity Growth Rates (Per Worker)
Sources: The solid-line gives the actual worker productivity growth rate (see Table 2). The dashed-line gives the simulated growth rate.

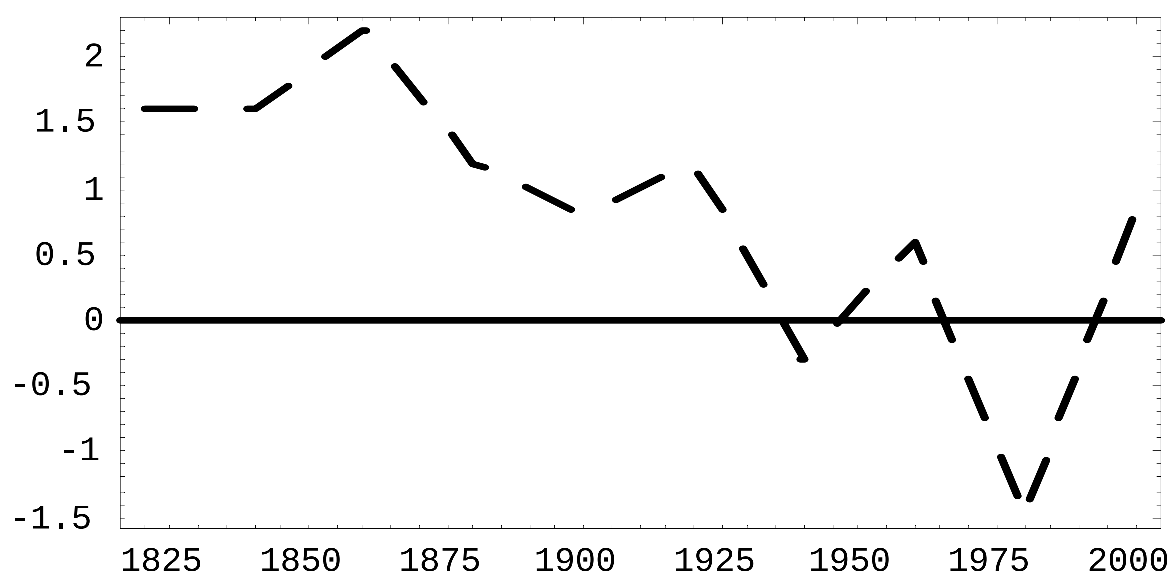

Figure 6 Predicted Growth Rate of Relative TFP (Traditional Sector)

TABLE 1: Growth Rate in Output per Hour-Worked

1820	0.39
1840	0.39
1860	0.56
1880	1.06
1900	1.53
1920	2.00
1940	2.31
1960	2.52
1980	1.62
2000	1.43

Notes: Growth rate is the annualized growth rate over the previous 20 year period. Growth rates are presented as percentage points.
Sources: Growth rates from 1800 to 1989 are based on Abramovitz and David (2001, Table 1 II). Growth rates from 1989 to 2000 are based on Gordon (1989) and Ferguson and Washer (2004).

TABLE 2: Growth Rate in Output per Worker

1820	0.31
1840	1.82
1860	1.32
1880	1.84
1900	1.53
1920	1.40
1940	1.72
1960	2.45
1980	1.58
2000	1.62

Notes: Growth rate at each date is the annualized growth rate over the previous 20 year period. Growth rates are presented as percentage points. *Sources:* 1800-1840 from David (2005, Table 6, Column 1 and Table 2.1, Column 3), 1840 to 1900 from Broadberry and Irwin (2006, Tables A1 and A2), 1900-1940 from Kendrick (1961, Table A XXII), 1940-1960 from Kendrick (1973, Table A 19) and 1960-2000 from *Economic Report of the President* (2006, Table B 2 and Table B 36)

TABLE 3: Fraction of Young Households with Land

1800	0.89
1820	0.68
1840	0.55
1860	0.67
1880	0.45
1900	0.25
1920	0.21
1940	0.07
1960	0.05
1980	0.01
2000	0.01

Sources: Entries are computed from (16). Fraction of labor in agriculture is from . Number of farms is from *Historical Statistics of the United States: Colonial Times to 1970* and Caplow et al (2002). The number of agricultural workers is from Lebergott and David (1967), Kendrick (1969), and the Economic Report of the President. The ratio of farms to agriculture labor in 1820 and 1840 were assumed to be equal to the ration in 1850.

Table 4 Calibrated Parameters and Initial Conditions

T	0.5000
τ	0.1600
γ	0.2250
θ	0.5110
ε	0.5403
ψ	0.2693
β	0.4136
α	0.4000
ρ	0.3000
ϕ	0.1000
δ	1.0000
$r_{1800}(annualized)$	0.1000
$\overline{e}$	0.0800
e_{1780}	0.0817
f	0.6171
$g(annualized)$	0.0033

Table 5 Growth Accounting

	Growth Rate	% due to $A^{\frac{1}{1-\alpha}}$	% due to k^{α}	% due to h	% due to $\bar{m}$
1800-1900					
y^{hour}	1.03	54	2	12	31
y^{worker}	1.46	38	1	8	51
1900-2000					
y^{hour}	1.70	33	21	26	20
y^{worker}	1.83	31	19	25	25
1800-2000					
y^{hour}	1.37	41	14	20	24
y^{worker}	1.64	34	12	17	37