

SOME VIEWS OF RUSSELL AND RUSSELL'S LOGIC BY HIS CONTEMPORARIES,
WITH PARTICULAR REFERENCE TO PEIRCE

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In the years prior to World War I, it was not yet definitively certain that the new logic developed and espoused by Frege and Russell was the “wave of the future” of mathematical logic. Ernst Schröder’s *Vorlesungen über die Algebra der Logik* (1895–1905) was still new and viewed as the cutting edge of (algebraic—as opposed to Aristotelian) logic; Giuseppe Peano still proclaimed, in his earliest work on logic, his indebtedness to George Boole, Charles Peirce, and Ernst Schröder, in the first footnote of his *Arithmetices principia nova methodo exposita* (1898), while, in a letter to Russell of 19 March 1901,¹ Peano told Russell that his paper “Sur la logique des relations avec des applications à la théorie des séries” on the logic of relations (1901) just “fills a gap between the work of Peirce and Schröder on the one hand and” his own “*Formulaire* on the other;” Alfred North Whitehead’s *Treatise on Universal Algebra* (1898) also still new, brought together the algebraic logic of Boole with the linear and multilinear algebras of Arthur Cayley, James Joseph Sylvester, William Rowan Hamilton, and Charles Peirce’s father Benjamin Peirce, and the calculus of extension of Hermann Grassmann. In his *Treatise*, Whitehead (p. x) not only named Hamilton and De Morgan as “the first to express quite clearly the general possibilities of algebraic symbolism,” but continually expressed his indebtedness to Boole, Grassmann, De Morgan, Schröder, and John Venn; and he described his *Treatise* (p. v) as providing a “thorough investigation of the various systems of Symbolic Reasoning allied to ordinary Algebra,” the “chief examples” of which were Hamilton’s Quaternions, Grassmann’s Calculus of Extension, and Boole’s Symbolic Logic. Cambridge University logician William E. Johnson, asked in 1905 to evaluate Whitehead’s work, wrote that Whitehead’s contributions to Boolean algebra and algebraic logic, although yielding “remarkable results and in a manner exhibiting extraordinary power” and “giving new life to the study of symbolic logic,”² nevertheless did not receive the attention which it deserved. Edward Huntington (1933, p. 278), Charles Peirce’s one-time correspondent, went further, seeing Whitehead’s work as the culmination and apex of the work in algebraic logic of the last half of the 19th century, asserting that algebraic logic, and in particular Boolean algebra and universal algebra was “origi-

nated by Boole, extended by Schröder, and perfected by Whitehead.” Russell’s British colleague Arthur Thomas Shearman, writing at the start of the 20th century on the development of symbolic logic in a work bearing that title (1906) of the last half-century, concentrating on the work of the British logicians, saw the work of Frege and Russell as minor natural extensions (and slight refinements) of the work of Boole, Augustus De Morgan, and William Stanley Jevons. Paul Carus, editor of *The Monist*, shared the view (1910, p. 54) that Schröder, Peirce, Peano, Russell and Couturat all belonged to the line of workers who sought to broaden traditional logic by their attempts to “transfer the accomplishments of mathematics upon logic.” Couturat himself asserted (1904, pp. 129–130) that Russell’s *Principles* was essentially “*une systématisation et une synthèse*” of the work of Russell’s predecessors, most notably Peano, Whitehead, Schröder, and Russell himself. If one read the published comments in the decade between publication of Russell’s *Principles* (1903) and Whitehead and Russell’s *Principia Mathematica* (1910–1913), one would in all likelihood be brought to conclude that the work of Peirce and Schröder was still at the forefront of logical research, and that it would, in the foreseeable future, be the point of departure for continued research.³

An examination of archival documents of Charles Peirce, Christine Ladd-Franklin, and others shows a tendency for strong denigration of Russell and his work in logic during the years 1903 to 1913 among many of Russell’s contemporaries, especially, but not exclusively his older contemporaries. Ladd-Franklin, for example, wrote a note in which she complains that Russell (and Whitehead) wrote as if Peirce and Schröder had never existed, clearly insinuating that Russell plagiarized the work of Peirce and Schröder, while Peirce for his part found Russell’s work “nauseating,” and philosopher William James bluntly called Russell “ass,” saying he would prefer being “a-logical, if not illogical” than adopt Russell’s techniques.⁴

Not until after the close of World War I, and thus roughly five years after the third volume of Whitehead and Russell’s *Principia Mathematica* appeared, that is, beginning in 1918, do we clearly notice the adoption of the estimation by the community of historians and philosophers of logic at large that the logic of Frege and Russell, and particularly the logical system of the *Principia*, is superior to, has surpassed, and ought to replace, the classical Boole-Schröder calculus, and that the latter is limited and in some measure obsolescent, if not obsolete. Thus it is in 1918 that Clarence Irving Lewis who, even in his solid exposition and historical outline of the development of the classical Boole-Schröder calculus, wrote of that calculus as a “classic” in the sense that it was becoming an antique.⁵ Contemporaneously, Georg Behrens wrote in *Die Prinzipien der mathematischen Logik bei Schröder, Russell und König* (1917; 1918, pp. 9–10) that

Schröder's *Vorlesungen über die Algebra der Logik* is a "special mathematical field which leans heavily upon logic," whereas Russell studied mathematical deduction and, unlike Schröder, "developed a presentation of the logical calculus." Borrowing a distinction which was used by the late Jean van Heijenoort (1967a), Schröder, like Boole, De Morgan, Peirce, and the algebraic logicians of the second half of the 19th-century, developed a *logica utens*, a specialized logic restricted to a specific universe of discourse, whereas Russell—and Frege—devised a one true logic, a *logica magna*, encompassing the entire universal domain, so that there is only one logic, the mathematical logic of *Principia*. And thus the classical Boole-Schröder calculus is reduced, depending upon one's perspective, to either a primitive, preparatory stage on the way to mathematical logic, or a specialized case of mathematical logic, the class calculus, or to a mere interesting but minor sidelight, or even a dead-end, in the development of mathematical logic, rather than what it was to its practitioners—the current state of the art of mathematical logic.⁶ In writing these lines, Behrens cited the "Summary" of *4 at p. 114 in the *Principia Mathematica* that "symbolic logic considered as a calculus had undoubtedly much interest on its own account; but in our opinion this aspect has hitherto been too much emphasized, at the expense of the aspect in which symbolic logic is merely the most elementary part of mathematics, and the logical prerequisite of all the rest."⁷

In apparent reply to this Russellian claim, but actually *circa* 1897, in an untitled manuscript on Schröder's logical algebra (MS 524:4–5), and, we might add, posthumously as well to Behrens' interpretation of it in contrast with Schröder's goal in treating the logic of relations in the *Vorlesungen*, Charles Peirce wrote that Schröder developed a calculus which "embraces all ordinary formal logic as nothing but an egregiously simple case." Peirce immediately continued: "The logic of relations is, therefore, far from being a specialized branch of logic. On the contrary, it greatly enlarges and amplifies all logical conceptions..." (MS 524:2–4). For Peirce and logicians like him at the end of the 19th century, then, the algebraic logic that they developed was, in the sense enumerated, a *logica magna*. Peirce's remarks, although predating the statement of Whitehead and Russell and the statement of Behrens, as well as all those who thereafter adopted this "Russellian" conception, was typical of attitudes towards the algebraic logic of the 19th- and early 20th-century of those who were working in the field until that time, and even up until the late 1910s. In that sense, Peirce's statement may be said to be his anachronistic and posthumous reply to Russell, Behrens, and the followers of Russell-Fregean logic. Peirce's views of the late 1890s were echoed even after the publication of Russell's *Principles of Mathematics* in 1903, and if it was not the universal opinion among logicians of that day, it remained prevalent among Peirce's associates and followers. Thus, for exam-

ple, in their article of 1905 on “Symbolic Logic”, Edward Huntington and Christine Ladd-Franklin wrote (at p. 1) that “Symbolic Logic, or Mathematical Logic, or the Calculus of Logic,— called also the Algebra of Logic (Peirce), Exact Logic (Schröder), and Algorithmic Logic or Logistic (Couturat), — covers exactly the same field as Formal Logic in general...” and that these terms are quite synonymous. The implication too is that Russell’s contributions in the *Principles* are a contribution to the same logic to which Boole, De Morgan, Peirce, and Schröder had made contributions.

Notable among Russell’s early contemporaries who, in the last years of the 19th- and first years of the 20th-century challenged Russell’s view of the primacy of Russell’s work in logic, and even of Russell’s self-appraisal of his contributions to logic, were Louis Couturat, Charles Peirce’s erstwhile student Christine Ladd-Franklin, Charles Peirce himself, and Charles Peirce’s friend and close ally, philosopher William James. Their comments on Russell and his work, both published and private, clearly illustrate that, in the view of many of Russell’s early contemporaries in that period between the publication of Russell’s *Principles of Mathematics* and of Whitehead and Russell’s *Principia Mathematica* (1910-13), and even for several years thereafter, Russell’s conception of logic not only as was not universally or immediately adopted, but that logicians of the period indeed often enough dealt particularly harshly with Russell and his work in logic.

Some salient illustrations of these negative reactions towards Russell and his work in logic, therefore, follow.

Charles Sanders Peirce and many his friends, students, and colleagues held that Russell held an exaggerated notion of his own importance and in particular of his contributions to logic. Many, including Peirce’s erstwhile Johns Hopkins University student Christine Ladd-Franklin, even went so far as to imply that Russell was plagiarizing the work of Peirce and Schröder. Neither she, nor Peirce, nor any of Peirce’s supporters would have been aware of Russell’s self-appraisal of his contributions, made in a letter to Ottoline Morrell dated 21 August 1912, in which Russell, referring to the *Principles of Mathematics*, wrote⁸ that “mathematical philosophers have different thoughts from what they w[oul]d have if I had not existed.” Without of course knowing of this assertion of Russell’s, Harvard philosopher, logician, and Peirce correspondent Josiah Royce, reminiscing with Victor Lenzen about the spring of 1914,⁹ complained that Russell had “received more attention than any logician since Aristotle.”

Of all the early contemporaries of Russell, Charles Sanders Peirce, whose work was broadly slighted and badly treated by Russell, had some of the harshest judgments amongst Russell’s elders. While Russell, as we shall see momentarily, sought to take credit for having created the logic of relations, Peirce, in the third of his Lowell lectures

of 1903 wrote (MS 459:20), with respect to Whitehead's "On Cardinal Numbers" (1902) to which Russell contributed and which deals with elementary set theory in terms of the logic of relations, presented in Peaneque notation, and to Russell's *Principles of Mathematics* (1903), that:

...quite recently Mr. Whitehead and the Hon[orable] Bertrand Russell have treated of the subject; but they seem merely to have pre[sented] truths already known into a uselessly technical and pedantic form.

Peirce also thought that, in contrast, he did a much better job of the enterprise, declaring (as published in the Hartshorne & Weiss edition of his *Collected Papers* (1934, p. 91)) that:

My analyses of reasoning surpasses in thoroughness all that has ever been done in print, whether in words or in symbols—all that De Morgan, Dedekind, Schröder, Peano, Russell, and others have done—to such a degree as to remind one of the differences between a pencil sketch of a scene and a photograph of it.

We know that Russell acquired a copy of Schröder's *Vorlesungen über die Algebra der Logik* in September 1900, and took notes on both that work and on Schröder's *Der Operationskreis des Logikkalkuls* (1877) and "Sur une extension de l'idée d'ordre" (1901) in 1901,¹⁰ as well as on Charles Peirce's "On the Algebra of Logic" (1880) and "On the Algebra of Logic: A Contribution to the Philosophy of Notation" (1885) in 1900-1901. In the *Principles* (p. 10), Russell called Schröder's *Algebra* "the most complete account of non-Peanesque methods," thus seeming to agree with Peirce that the Boole-Schröder calculus, if not Peirce's own treatment and presentation of it, is by far the best and most advanced logic up to that time, excepting perhaps Peano's notational rendition; he would certainly *seem* to also thus have agreed with Peirce's assessment *circa* 1897 of the *Algebra*, in the manuscript titled "Schröder's Logic of Relations" (MS 521:12), that "Prof. Schröder's work is, and must for many years remain, the standard treatise upon exact logic...." But we know, of course, that Russell would *not* have agreed with Peirce on this latter score.

In any event, Russell had little else, and still less explicitly, to say about the contributions either of Peirce or of Schröder. Indeed, he wrote (in 1946, p. xv) that he "read nothing of him [Peirce] until 1900, when I became interested in extending symbolic logic

to relations, and learnt from Schröder's *Algebra der Logik* that Peirce had treated of the subject."¹¹ This fully accords with the dates of Russell's reading notes of Peirce and Schröder; yet it seems a remarkable confession, in light of Huntington's retrospective remark (1933, p. 278) that algebraic logic and universal algebra were "...extended by Schröder, and perfected by Whitehead."

Russell, however, dismissed the work of Peirce and Schröder as negligible for the development of his own treatment of the logic of relations, saying that he had already completed his work in that field before he read theirs, telling French philosopher and logician Louis Couturat, for example, in a letter of 2 June 1903 (R57 = R58) that he read Schröder's work only after learning of Peano, and that "it is not therefore essential to go through him." This accords less well with Russell's earlier recommendation to Couturat, in a letter dated 11 February 1899, that Couturat might find Peirce's *Studies in Logic* (1883) of some interest. Russell went so far as to claim, in a letter to Philip Jourdain of April 1910 that it was he who "invented" "his" Logic of Relations during September 1900.¹² He also asserted in a letter to Helen Thomas dated 31 December 1900 that in October of that year, preparing what became the *Principles*, that in writing it he "invented a new subject, which turned out to be all mathematics for the first time treated in its essence."¹³

How much, how carefully, and with what understanding, Russell read the works of Peirce and Schröder, have been matters of speculation.¹⁴ Christine Ladd-Franklin could, of course, not know what Russell read, or when he read it. But being familiar with Peirce's and Schröder's work as well as with Russell's *Principles*, was in a good position to compare them. In undated notes probably prepared for a Columbia University lecture class, *circa* 1913, she wrote:

It should now be clear how the logic of *Principia* is related to the logic we have presented, following the materials of Peirce and Schröder.... But Whitehead and Russell plainly 'imply' that P[eirce] and S[chröder] were absolutely non-existent!

Charles Peirce held Russell's treatment of relations to be incomplete; in a marginal notation for p. 24, lines 25–28 in his copy of *Principles*, for example, Peirce wrote: "He considers only dyadic relations." Most of Peirce's marginal notations and comments on Russell concern Russell's published criticisms of his [Peirce's] ostensible failure to distinguish types of collections and between class inclusion and material implication.¹⁵ It was in connection with a discussion of Cantor's "Beiträge zur Begründung der transfi-

niten Mengenlehre” (1895-1897), of Cantor’s distinction between complete and incomplete totalities and between implication and class inclusion that Peirce wrote (MS 459:19–20) for his third Lowell lecture in 1903 that “...quite recently Mr. Whitehead and the Hon. Bertrand Russell treated of the subject...” In the fifth of his Lowell Lectures (1903), Peirce wrote (MS 469:20) that what “puzzles the Hon. Bertrand Russell in his ‘Principles of Mathematics’ is whether a collection which has but a single individual member is identical with that individual,” and he attributes Russell’s puzzlement to his failure to make appropriate terminological and conceptual distinctions between these. Peirce himself emphasized to Cantor, in letters dated 21 December 1900 and 23 December 1900,¹⁶ the need to distinguish between sets and classes. It would seem that Russell’s criticisms of Peirce’s failure to distinguish between class inclusion and set membership was, if not entirely unfair, then at least disingenuous; for Peirce used the same symbol for both, as well as for other relational connectives, but made clear from the context in which he was working which relation he had in mind. The gist of Peirce’s marginalia to his copy of the *Principles* was that Russell’s difficulties with Peirce’s and Schröder’s ostensible lack of proper distinctions was rooted in Russell’s own failure to distinguish material implication and truth-functional implication (conditionality), and in Russell’s erroneous attempt to treat classes, in function-theoretic terms, as individual entities.

Russell’s criticisms of Peirce as failing to distinguish set membership from class inclusion, first found in Russell’s “Sur la logique des relations avec des applications à la théorie des séries” (1901), is revived, in slightly different guise, in the *Principles*, where Russell asserts at once (1903, p. 24) that, while Peirce and Schröder admittedly “realized the great importance of the subject” of the logic of relations, also nonetheless “unfortunately their methods, being based, not on Peano, but on the older Symbolic Logic derived (with modifications) from Boole, are so cumbrous and difficult that most of the applications which ought to be made are practically not feasible,” and he then levels the criticism (1903, p. 24) that: “In addition to the defects of the old Symbolic Logic, their method suffers technically...from the fact that they regard a relation essentially as a class of couples, thus requiring elaborate formulæ of summation for dealing with single relations.” In a letter to Philip Jourdain of 15 April 1910, Russell went so far as to call Schröder’s methods “hopeless.”¹⁷ In reply to Russell’s assertion (1903, p. 26) that

Peirce and Schröder consider what they call the relative sum of two relations R and S , which holds between x and z , when, if y be any other term whatever, either x has y to the relation R , or y has to z the relation S . This is a complicated notion, which I have found no occasion to

employ, and which is introduced only in order to preserve the duality of addition and multiplication,

Peirce in a letter to Victoria Welby of 12 October 1904, stated that:¹⁸

As to my algebra of dyadic relations, Russell in his book which is superficial to nauseating to me, has some silly remarks about my “relative addition” etc., which are mere nonsense. He says, or Whitehead says, that the need for it *never* occurs if you bring in the same mode of connection in any other way. It is part of a system which does not bring in the mode of connection in any other way. In that system it is indispensable. But let us leave Russell and Whitehead to work out their own salvation.

Russell was made aware of Peirce’s remark by Welby. Welby, who sought to promote a dialogue between Russell and Peirce, and so must have passed Peirce’s letter along to Russell, since Russell mentions it on the first page of his letter to Welby of November 14, 1904. Russell responds, in a letter to Welby of 27 December 1904 that he “does not know where Whitehead or I have said that the need of Dr. Peirce’s Algebra of dyadic relations seldom occurs,” and adds that he thinks that “a symbolism based on Peano’s is practically more convenient, but I hold it quite essential to have a method of expressing relations, & I have always thought very highly of Dr. Peirce for having introduced such a method.”

Given numerous opportunities to respond directly to Russell in print, Peirce proved reluctant to do so. In private, his remarks concerning Russell and Russell’s work in logic were negative. Asked to review the *Principles* for the journal *Science*, Peirce delayed doing so for over a year; he told Christine Ladd-Franklin in a letter of 27 July 1904 (MS L237:27) that: “...a year has passed since I agreed to notice Russell’s vol. I, and I feel its pretentiousness so strongly that I cannot fail to express it in a notice.” In a similar vein, he wrote to Australian philosopher F. William Frankland on May 8, 1906 that:¹⁹ “In my opinion Russell and Whitehead are blunderers constantly confusing different questions.” In a letter to Welby of 1 December, 1903, Peirce concluded, from a cursory reading of the *Principles*, that “whatever merit it may have as a digest of what others have done, it is pretentious and pedantic—attributing to its author merit that cannot be accorded him.”

Here is the first suggestion that the *Principles* was not to be esteemed an original work, the first suggestion of possible plagiarism. The theme of Russell's lack of originality will be found elsewhere as well.

Christine Ladd-Franklin also raised the question of Russell's lack of originality, more forcefully than did Peirce himself; she wrote, as already cited, that Russell and Whitehead "plainly 'imply' that P[erice] and S[chröder] were absolutely non-existent!" Josiah Royce and Fergus Kernal (1916, pp. 706–707) have likewise asserted, albeit somewhat less blatantly suggesting plagiarism, concerning "...Peirce's researches on the algebra of logic, and in particular the logic of relatives," that "many of the most recent researches, including those of Bertrand Russell, are still due to his," Peirce's, "influence, although Russell, ...has a somewhat inadequate sense of his own generally indirect indebtedness to Peirce's work in this field."

It would not be difficult to find examples of Russell's failure to assign proper credit. The so-called "principle of reduction" given as axiom 10 in the *Principles* (§18, p. 17), for example, according to which $((p \supset q) \supset p) \supset q$ (when $p \supset p$ and $q \supset q$), is exactly the "fifth icon" which is to be found in Peirce's "On the Algebra of Logic: A Contribution to the Philosophy of Notation" (1885, p. 189), and which we today call Peirce's Law.

Ladd-Franklin was among those who sought to prod Peirce into replying to Russell; thus she wrote in a letter to Peirce of 24 July 1904:

Do tell me how it strikes you—all this recent work of Bertrand Russell, Peano, Couturat & their school, which they make so much of. Don't you think that they exaggerate both its originality & its importance? Are you not going to write something on the subject?

Similar requests were to be had from Eliakim Hastings Moore, editor of the American Mathematical Society's *Transactions*, who on 14 October 1902 wrote to Peirce, asking him to respond to Whitehead's article "On Cardinal Numbers" (1901), and in particular to compare his own work with the work of Whitehead and Russell, noting that Whitehead and Russell wrote in Peano's notation and used "Russell's additions on the algebra of relations in general. ...It would give me much pleasure if you would let me know what you think concerning Russell's work, especially in comparison with your own...." Peirce failing to respond, Moore again asked Peirce, in a letter of 31 December 1903, what he thought of Russell's *Principles of Mathematics*.

When, finally Peirce did write on the *Principles*, it was in conjunction with Welby's *What is Meaning?* (1903), in which Peirce lightly dismissed the *Principles*, merely acknowledging that "the severe and scholastic labors" that went into it "bespeaks a grit and industry, as well as a high intelligence," while adding that: "Whoever wishes a convenient introduction to the remarkable researches into the logic of mathematics that have been made during the last sixty years...will do well to take up this book." Peirce's remarks, he confessed to Welby, were intended to serve as a contrast between the two books, and to strongly hint that the *Principles* was quite unoriginal.²⁰ Not surprisingly, in a letter to William James of 25 December 1909, Peirce wrote of his own work on the logic of relatives that it "simply revolutionizes Logic" and that it "ought to be the Logic of the Future."²¹

Louis Couturat, more nearly Russell's age and early on one of Russell's staunchest supporters and best interpreters for French philosophers, and one of those not associated personally with Peirce, in his *L'algèbre de la logique* (1905; English translation: 1914, p. 92), wrote that the algebra of logic "ought...to develop into a logic of relations, which LEIBNIZ foresaw, which PEIRCE and SCHRÖDER founded, and which PEANO and RUSSELL seem to have established on definite foundations," evidently thus sharing with Arthur Shearman, Giuseppe Peano, and Paul Carus, among many others, the view that the work of Russell, Peano, and Frege belonged within, and indeed was inseparable from, the work of the older algebraic logicians, that the logic that was to be found in works such as Russell's *Principles of Mathematics* was a refinement and continuation of the work that had led to, and was presented as the most contemporary state of, the classical Boole-Schröder calculus. I interpret Couturat's remark here to assert that Peano and Russell (along with Frege, we might add), set a foundation underneath the classical Boole-Schröder calculus, rather than replaced it with an altogether new symbolic logic. Elsewhere, Couturat felt compelled to explicitly warn Russell against the denigration of the Boole-Schröder calculus to the profit of Peano's mathematical logic, writing, in a letter to Russell dated 27 January 1901, that:

...one must not deprecate the Boole-Schröder system and sacrifice it to Peano: it has its goal and its use in pure logic just as Peano's has its in Math. And the theory of logical equations has great importance since all logical problems amount to a system of equations and inequations.

Russell nevertheless dismissed Schröder's work by telling Couturat, in a letter of 9 June 1903, that Schröder speaks "prose" without knowing it.

Norbert Wiener's assertions, in his doctoral dissertation for Harvard University in 1913, offering a comparison between the logical systems of Schröder's *Vorlesungen über die Algebra der Logik* and Whitehead and Russell's *Principia Mathematica*, in which Wiener asserts that essentially the two systems are equal in expressive power, and the discussions which Wiener had held with Russell about his claims, has already been treated by Ivor Grattan-Guinness (1975); moreover, Geraldine Brady (2000) has reproduced salient and extensive excerpts, including the introductory and concluding chapters, of Wiener's thesis, and I need not here repeat any but the main point: namely that Wiener regarded the two systems as essentially equivalent, and Russell's retort: that Wiener considered only "the more conventional parts of *Principia Mathematica*."²²

Cumulatively, the views of these logicians and others suggest that there was nothing particularly inevitable about the predominance which Russell or his conception of logic, or, for that matter, his conception of his ostensibly singular contributions to the development of symbolic logic, would come to enjoy in later years. For Peirce, *circa* 1897, for example, "Prof. Schröder's work [*Algebra*] is, and must for many years remain, the standard treatise upon exact logic..." (MS 521:12).

That Russell was aware of the criticisms of him by Peirce and Peirce's supporters is attested to by the fact that he expressed the hope, in his letter to Welby of 11 November 1904 (pp. 2–3), that the second volume of the *Principles* which he planned "will do much to persuade such opponents as Mr. C. S. Peirce" of the correctness of his own views and position.

One can quote a number of logicians and philosophers in the last years of the late 19th century and into the second decade of the 20th who found the work of Peirce and/or Schröder to be still at the forefront of logical research; among them William James, Edwin Bidwell Wilson, Josiah Royce, William Clifford, and others, while Peirce and Schröder formed something of a mutual admiration society.²³ Whether their *fin de siècle* assessments were correct or not, it is clear that there was little certainty or unanimity regarding the accomplishments of Russell, either in *The Principles of Mathematics*, nor yet even in his and Whitehead's *Principia Mathematica*, at the time they appeared, and that Peirce and Schröder were not yet easily dismissed as having fallen into oblivion or obsolescence.

Notes

¹ Quoted in (Kennedy 1975, p. 206).

² In his “Report to the Sub-Committee on Higher Degrees of the Cambridge University Special Board on Mathematics”, published in the *Cambridge Register*.

³ For example: E. B. Wilson wrote (1904, p. 76) that “Boole had freed us from Aristotelianism and that C. S. Peirce and Schröder had carried the technique of logic much further.” Maxime Bôcher wrote (1904, p. 119) that: “Fortunately, the mathematical logicians from Boole down to C. S. Peirce, Schröder, and Peano and his followers,” including, independently, Frege, “have been able to make a rather short list of logical conceptions and principles upon which it would seem that all exact reasoning depends.” Josiah Royce (as quoted in (Ketner 1987, p. 18)) asserted that “Mr. Charles Peirce has now been for many years the principal representative in this country of a type of investigation in Logic which seems to me, as a student of the subject, to be of very great importance.” William James (as quoted in (Ketner 1987, p. 20)) thought that Peirce was “in the very front rank of American thinkers...and his Logic when published will unquestionably...be recognized all over the world as an epoch-making work.” British mathematician and philosopher of science William Clifford (according to Edward L. Youmans, as reported by (Fisch 1986, p. 129)) called Peirce “the greatest living logician, and the second man since Aristotle who has added to the subject” of logic “something material, the other man being George Boole....” As late as 1922, Jan Lukasiewicz in his Inaugural lecture listed Peirce as one of the most prominent representatives of mathematical logic of the day.

⁴ Letter to Charles Peirce, 24 December 1909; quoted in (Perry 1935, p. 680).

⁵ See p. 118 of the 1960 abridged edition of his *Survey of Symbolic Logic* (1918).

⁶ How and why this turn in point of view occurred is subject for another discussion, one which I dealt with in part elsewhere—see, e.g., (Anellis and Houser 1991) and my (1995)—and intend to develop further. Here my purpose is to document the reactions of Russell’s early contemporaries to Russell and his work in logic, basing the illustrations in largest measure upon contemporary, and especially archival, materials.

⁷ Compare this with Jean van Heijenoort’s statement, in van Heijenoort’s “Preface” to *From Frege to Gödel* (van Heijenoort 1967b, p. vi) that although “Boole, De Morgan, and Jevons are regarded as the initiators of modern logic and rightly so,” and the period of their work “would, no doubt leave its mark on the history of logic, but would not count as a great epoch.”

⁸ As quoted in (Clark, p 189) and (GarciaDiego, p. 132).

⁹ As reported by Lenzen (1966, p. 4).

¹⁰ See my (1990/1991) for a sketch of Russell’s notes on Schröder.

¹¹ It may or may not completely square with the evidence, from Russell’s own reading log for 1891-1902, that he read Whitehead’s *Treatise* almost as soon as it appeared, in March 1898, and should, had he read it, have found Whitehead’s expressions (1898, p. x) of indebtedness “in regard to Symbolic Logic to Boole, Schröder, and Venn.”

¹² Quoted in (Grattan-Guinness 1977, p. 133).

¹³ Quoted in (Griffin 1992, p. 207, letter 91).

¹⁴ Benjamin Hawkins, for example, thinks (1992, pp. 43–44; 1997, 137) that Russell actually read very little of Peirce’s work on logic, totaling the equivalent of approximately fourteen pages in the third volume of Peirce’s *Collected Papers* (1933), merely skimming Peirce’s “On the Algebra of Logic” (1880) and “On the Algebra of Logic: A Contribution to the Philosophy of Notation” (1885). Hawkins says that these “few pages...represent Russell’s reading of Peirce” on logic, and supposes that that may, at least partially, explain Russell’s inaccuracies regarding Peirce.

¹⁵ Peirce’s marginal annotations in his copy of Russell’s *Principles* are littered with comments such as “not so” (p. 12, ll. 12–13), “utterly false” (p. 13, end ¶13), and “Ridiculous modes of formulation” (p. 16, end ¶18).

¹⁶ Published in (Peirce 1976, III/2), pp. 767–771, 772–779 respectively.

¹⁷ Quoted in (Grattan-Guinness 1977, p. 134).

¹⁸ See p. 30 in (Hardwick 1977, pp. 22–35).

Peirce’s supposition that Whitehead may have had a hand in the writing of the *Principles* may be attributed to his knowledge that Russell contributed the section on ordinal numbers to Whitehead’s article “On Cardinal Numbers” (1901) and to both Whitehead’s profuse expressions of indebtedness to Russell in that article and Russell’s profuse expressions of indebtedness to Whitehead in the *Principles*.

¹⁹ Quoted in (Peirce 1977, III/2, p. 78).

²⁰ Quoted in (Hardwick 1977, p. 9).

²¹ Quoted in (Peirce 1976, III/2, pp. 867–877), see esp. pp. 873–874.

²² See (Grattan-Guinness 1975, p. 130).

²³ See, e.g. (Houser 1990/1991) on Peirce and Schröder as seen through their correspondence. For other contemporary (early 20th-century) expressions of opinion regarding the classical Boole-Schröder calculus and of Peirce’s stature and contributions to mathematical logic, see note 3.

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